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CHARACTERIZATIONS OF *F*-PRIME IDEALS IN POSETS

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Abstract

In this article, we look at the ideas of f-prime ideals and f-semi-prime ideals of posets, as well as the many features of f-primeness and f-semi-primeness in posets. Classifications of f semi-prime ideals in posets are derived, as well as representations of a f-semi-prime ideal to be f-prime. Furthermore, the f-prime ideal separation theorem is addressed.

Keywords: poset, semi-ideals, f-prime ideal, f-semi prime ideal, m-system.

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1. Introduction

The concept of prime ideal, which arises in the theory of rings as a generalization of the concept of prime number in the ring of integers, plays a crucial role in that theory, as one might assume given the primes' fundamental place in arithmetic. Radicals play an important role in algebraic structures. The Jacobson radical is the intersection of all maximum ideals with unity in a commutative ring, whereas the ring's prime radical is the intersection of all prime ideals. The radical concept was utilized to launch the primary ideal, which was established on prime ideal principles.

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Van der Walt [18] defined s-prime ideals in non-commutative rings and deduced McCoy's [12] s-prime ideals discoveries. Several authors corroborated Van der Walt's earlier near-ring results. Murata *et al.* [14] proposed the concepts of f-prime ideals and f-prime radicals in ring theory in 1969, which generalized the concepts of prime ideals and prime radicals. Sardar and Goswami [17] expanded the principles and results of ring theory to semi-rings. Groenewald and Potgieter [7] developed f-prime in near-rings. Many authors studied f-prime ideals in various algebraic structures [1, 8]. The prime radical was described by Sambasiva Rao and Satyanarayana [16] in terms of highly nilpotent components of near-rings, and certain results of Hsu [10] were extended to f-prime and f-semiprime ideals in near-rings.

Several mathematical areas come across algebraic systems with partial or complete order. Many authors investigated various prime ideals of posets because the theory of partially ordered algebraic systems is critical.

Rav [15] proposed and investigated semi-prime ideals in lattices. If $a \wedge w \in H$ and $a \wedge v \in H$ jointly imply $a \wedge (w \vee v) \in H$, an ideal H of a lattice $\mathbb{L}$ is defined as semi-prime.

Following [15], Kharat and Mokbel [11] presented the concept of a semi-prime ideal in posets and explored various semi-primeness aspects, as well as defined the relationship between primeness and semi-primeness in posets. Because prime ideals and semi-prime ideals are used to describe specific classes of lattices, it is necessary to generalise and investigate these ideas for posets.

Catherine and Elavarasan [4] studied the notion of primal ideals in a poset and the relationship among the primal ideals and strongly prime ideals is considered. Catherine [6] discussed about strongly prime radicals and primary ideals of posets.

As a result, in this article, we have enlarged the fundamentals of prime ideals and semi-prime ideals to f-prime ideals and f- semi-prime ideals in posets. In addition, we obtained the condition for an ideal to be f-prime ideals in a poset. Also, f-prime ideals in a poset are characterized.

2. Preliminaries

Throughout this paper $(\mathbb{Q}, \leqslant)$ denotes a poset with smallest element 0. We refer to [9] and [11] for basic concepts and notations of posets. For $S \subseteq \mathbb{Q}$, $(S)^\ell = \{q \in \mathbb{Q} : q \leqslant s \text{ for all } s \in S\}$ indicates the lower cone of S in $\mathbb{Q}$ and $(S)^u = \{q \in \mathbb{Q} : s \leqslant q \text{ for all } s \in S\}$ indicates the upper cone of S in $\mathbb{Q}$. For all subsets S, T of $\mathbb{Q}$, we represent $(S,T)^\ell$ rather than $(S \cup T)^\ell$ and $(S,T)^u$ instead of $(S \cup T)^u$. For a finite subset $S = \{s_1, s_2, \ldots, s_n\}$ of $\mathbb{Q}$, we write $(s_1, s_2, \ldots, s_n)^l$ instead of $(\{s_1, s_2, \ldots, s_n\})^l$ and dually. Clearly for a subset S of $\mathbb{Q}$, $S \subseteq (S)^{u\ell}$

and $S \subseteq (S)^{\ell u}$. If $S \subseteq T$, then $(T)^{\ell} \subseteq (S)^{\ell}$ and $(T)^u \subseteq (S)^u$. Also, $(S)^{u\ell u} = (S)^u$ and $(S)^{\ell u \ell} = (S)^{\ell}$.

Following [19] and [20], a subset $B(\neq \emptyset)$ of $\mathbb{Q}$ is termed as semi-ideal if $q \in B$ and $s \leqslant q$, then $s \in B$. Also B is referred as ideal if $s, d \in B$ implies $(s, d)^{u\ell} \subseteq B$[9]. For ideals B_i of $\mathbb{Q}$, $\bigcap_i B_i$ is an ideal of $\mathbb{Q}$. However, $\bigcup_i B_i$ is not needed to be an ideal of $\mathbb{Q}$ in general. A semi-ideal (respectively, ideal) B of $\mathbb{Q}$ is referred as prime if $(s, d)^{\ell} \subseteq B$ implies either $s \in B$ or $d \in B$ [9].

An ideal B of $\mathbb{Q}$ is termed as semi-prime if $(r, s)^{\ell} \subseteq B$ and $(r, t)^{\ell} \subseteq B$ together imply $(r, (s, t)^u)^{\ell} \subseteq B$ for all $r, s, t \in \mathbb{Q}$[11]. For $s \in \mathbb{Q}$, the principal ideal (respectively, filter) of $\mathbb{Q}$ generated by s is $(s] = (s)^{\ell} = \{q \in \mathbb{Q} : q \leqslant s\}$ (respectively, [s)= $(s)^u = \{q \in \mathbb{Q} : q \geq s\}$). A subset $S(\neq \emptyset)$ of $\mathbb{Q}$ is known as an up directed set if $S \cap (r, s)^u \neq \emptyset$ for all $r, s \in \mathbb{Q}$.

Considering [4], an ideal J of $\mathbb{Q}$ is termed as strongly prime if $(I_1^*, I_2^*)^{\ell} \subseteq J$ implies either $I_1 \subseteq J$ or $I_2 \subseteq J$ for different proper ideals I_1, I_2 of $\mathbb{Q}$, where $I_1^* = I_1 \backslash \{0\}$. An ideal I of $\mathbb{Q}$ is called strongly semi-prime if $(A^*, B^*)^{\ell} \subseteq I$ and $(A^*, C^*)^{\ell} \subseteq I$ together imply $(A^*, (B^*, C^*)^u)^{\ell} \subseteq I$ for different proper ideals A, B and C of $\mathbb{Q}$.

A subset $N(\neq \emptyset)$ of $\mathbb{Q}$ is referred as a m-system if for $t_1, t_2 \in N$, there exists $t \in (t_1, t_2)^{\ell}$ such that $t \in N$. A subset $N(\neq \emptyset)$ of $\mathbb{Q}$ is termed as strongly m-system if for different proper ideals I_1, I_2 of $\mathbb{Q}$, whenever $I_1 \cap N \neq \emptyset$ and $I_2 \cap N \neq \emptyset$ imply $(I_1^*, I_2^*)^{\ell} \cap N \neq \emptyset$. It is obvious that for any ideal I_1 of $\mathbb{Q}$, $\mathbb{Q} \backslash I_1$ is a strongly m- system of $\mathbb{Q}$ if and only if I_1 is strongly prime. Every strongly m-system of $\mathbb{Q}$ is also a m-system of $\mathbb{Q}$. However, the converse is not always true in many cases; see Example 4.

Example 1. Consider $\mathbb{Q} = \{0, r, s, t, u, v\}$ and a relation $\leqslant$ defined on $\mathbb{Q}$ as follows.

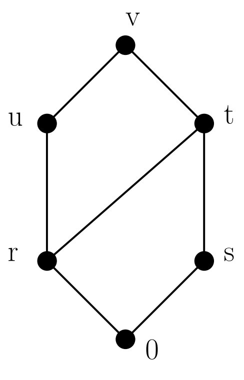

Figure 1. Example of prime ideal which is not strongly prime.

Then $(\mathbb{Q}, \leq)$ is a poset and $I = \{0, r, u\}$ is a prime ideal of $\mathbb{Q}$, but not strongly prime, since for ideals $A = \{0, s\}$ and $B = \{0, r, s, t\}$ of $\mathbb{Q}$, we have $(A^*, B^*)^{\ell} \subseteq I$, but neither A nor B contained in I.

Example 2. Let $\mathbb{Q} = \{0, a, b, c, d\}$ and define a relation $\leq$ on $\mathbb{Q}$ as follows.

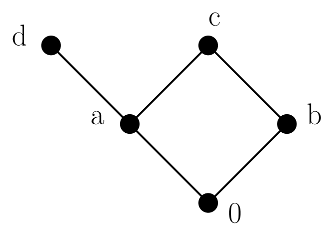

Figure 2. Example of semi prime ideal which is not strongly semi prime.

Then $(\mathbb{Q}, \leq)$ is a poset and $I = \{0\}$ is a semi prime ideal, but not strongly semi prime, since for ideals $A = \{0, a\}; B = \{0, b\}; C = \{0, a, b, c\}$ of $\mathbb{Q}$, we have $(A^*, B^*)^\ell \subseteq I$ and $(A^*, C^*)^\ell \subseteq I$, but $(A^*, (B^*, C^*)^u)^\ell = (a, c)^\ell = \{0, a\} \nsubseteq I$.

Every strongly prime ideal of $\mathbb{Q}$ is strongly semi prime ideal. But converse not true in general.

Example 3. Let $\mathbb{Q} = \{0, a, b, c, d\}$ and define a relation $\leq$ on $\mathbb{Q}$ as follows.

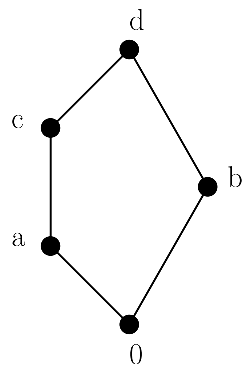

Figure 3. Example of strongly semi prime ideal which is not strongly prime.

Then $(\mathbb{Q}, \leq)$ is a poset and $I = \{0\}$ is a strongly semi prime ideal of $\mathbb{Q}$, but not strongly prime, for ideals $A = \{0, a, c\}, B = \{0, b\}$ of $\mathbb{Q}$, $(A^*, B^*)^\ell \subseteq I$, but $A \nsubseteq I$ and $B \nsubseteq I$.

Example 4. Consider $\mathbb{Q} = \{0, a, b, c, d, e\}$ and define a relation $\leq$ on $\mathbb{Q}$ as follows.

Then $(\mathbb{Q}, \leq)$ is a poset. Here $M = \{a, c, d\}$ is a m-system of $\mathbb{Q}$ which is not strongly m-system for $A = \{0, e, a\}$ and $B = \{0, e, a, b, c\}$, we have $A \cap M \neq \emptyset$ and $B \cap M \neq \emptyset$, but $(A^*, B^*)^\ell \cap M = \emptyset$.

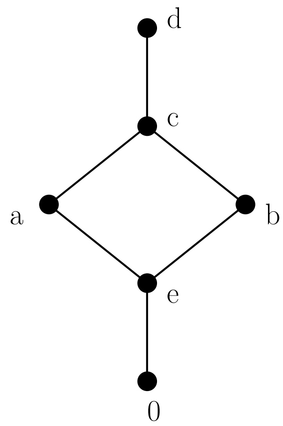

Figure 4. Example of m-system which is not strongly m-system.

3. f-PRIME IDEALS IN POSETS

For all element $q \in \mathbb{Q}$, we associate a unique ideal $f(q)$, which satisfies the following conditions:

(i) $q \in f(q)$ and

(ii) $x \in f(q)$ implies that $f(x) \subseteq f(q)$, for $x \in \mathbb{Q}$.

The collection of all such mappings from $\mathbb{Q}$ into set of all ideals of $\mathbb{Q}$ is indicated by $\mathbb{F}(\mathbb{Q})$.

Example 5. In a poset $\mathbb{Q}$, for each element q of $\mathbb{Q}$, if $f(q) = (q)^\ell$, the principal ideal generated by q, then it is obvious that f meets the preceding requirements.

Definition. For $f \in \mathbb{F}(\mathbb{Q})$, a subset S of $\mathbb{Q}$ is called an f-system if and only if it has a strongly m-system S_1 such that $S_1 \cap f(q) \neq \emptyset$ for each $q \in S$.

Definition. An ideal H of $\mathbb{Q}$ is called f-prime if and only if its complement H^c is a f-system of $\mathbb{Q}$.

It is clear that every strongly m-system is a f-system and every strongly prime ideal of $\mathbb{Q}$ is a f-prime ideal of $\mathbb{Q}$. But generally, the converse is not correct as shown in the below example.

Example 6. In the Example 1, consider a mapping f from $\mathbb{Q}$ into set of ideals of $\mathbb{Q}$ such that $f(0) = \{0\}$, $f(r) = \{0, r, u\}$,$f(s) = \{0, s\}$, $f(u) = \{0, r, u\}$, $f(t) = \{0, r, s, t, u, v\}$ and $f(v) = \{0, r, s, t, u, v\}$. Then $f \in \mathbb{F}(\mathbb{Q})$. Here $M_1 = \{u, v, t, r\}$ is a f-system and contains the strongly m-system $M_2 = \{u, v\}$, but M_1 is not a strongly m-system as for the ideals $D = \{0, r, u\}$, $H = \{0, r, s, t\}$, we have $D \cap M_1 \neq \emptyset$ and $H \cap M_1 \neq \emptyset$ with $(H^*, D^*)^\ell \cap M_1 = \emptyset$.

Remark 7. In Example 1, if we define a mapping f from $\mathbb{Q}$ into set of ideals of $\mathbb{Q}$ such that $f(0) = \{0\}$, $f(r) = \{0, r, s, t, u, v\}$, $f(s) = \{0, s\}$, $f(u) = \{0, r, u\}$, $f(t) = \{0, r, s, t, u, v\}$ and $f(v) = \{0, r, s, t, u, v\}$. Then $f \notin \mathbb{F}(\mathbb{Q})$.

Theorem 8. *For any f-prime ideal H of $\mathbb{Q}$, $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ implies that either $\eta_1 \in H$ or $\eta_2 \in H$ for different proper ideals $f(\eta_1)$, $f(\eta_2)$ of $\mathbb{Q}$, where $f(\eta_1)^* = f(\eta_1)\backslash\{0\}$.*

Proof. Suppose not, $\eta_i \in \mathbb{Q}\backslash H$ for $i = 1, 2$. As H is a f-prime ideal, we have $\mathbb{Q}\backslash H$ is a f-system. Then there exists a strongly m-system $M \subseteq \mathbb{Q}\backslash H$ such that $M \cap f(\eta_i) \neq \emptyset$ for $i = 1, 2$. As M is a strongly m-system of $\mathbb{Q}$, we get $(f(\eta_1)^*, f(\eta_2)^*)^\ell \cap M \neq \emptyset$ which implies $(f(\eta_1)^*, f(\eta_2)^*)^\ell \cap \mathbb{Q}\backslash H \neq \emptyset$, a contradiction. ■

Definition. An ideal H of $\mathbb{Q}$ is termed as f-semi-prime if $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq H$ together imply $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq H$ for different proper ideals $f(\eta_1), f(\eta_2)$ and $f(\eta_3)$ of $\mathbb{Q}$ and $f \in \mathbb{F}(\mathbb{Q})$.

Lemma 9. *The intersection of f-semi-prime ideals of $\mathbb{Q}$ is again a f-semi-prime ideal of $\mathbb{Q}$ for $f \in \mathbb{F}(\mathbb{Q})$.*

Proof. Let $H = \cap G_j$, where G_j 's are f-semi-prime ideals of $\mathbb{Q}$ and for different proper ideals $f(\eta_1), f(\eta_2), f(\eta_3)$ of $\mathbb{Q}$, $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq H$. Then $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq G_j$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq G_j$ for all j. Since each G_j is f-semi-prime ideal, we have $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq G_j$ for all j. So $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq \cap G_j = H$. ■

Theorem 10. *Let H be an ideal of $\mathbb{Q}$. If H is f-prime, then H is f-semi-prime.*

Proof. Let $f(\eta_1), f(\eta_2)$ and $f(\eta_3)$ be different proper ideals of $\mathbb{Q}$ under the mapping $f : \mathbb{Q} \to Id(\mathbb{Q})$ with $f \in \mathbb{F}(\mathbb{Q})$ such that $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq H$.

Case (i). If $\eta_1 \in H$, then $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq (f(\eta_1)^*)^\ell \subseteq H$.

Case (ii). If $\eta_1 \notin H$, then by the f-primeness of H, we have $\eta_2 \in H$ and $\eta_3 \in H$ which imply $((\eta_2, \eta_3)^u)^\ell \subseteq H$ for $\eta_2 \in f(\eta_2)^*; \eta_3 \in f(\eta_3)^*$, so $((f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq H$ and $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq ((f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq H$. ■

The example below shows that the contrary of Theorem 10 is not consistent with the prediction. That is, not every f-semi prime ideal of $\mathbb{Q}$ is a f-prime ideal of $\mathbb{Q}$.

Example 11. Consider $\mathbb{Q} = \{0, a, b, c, d\}$ and a relation $\leqslant$ defined on $\mathbb{Q}$ as follows.

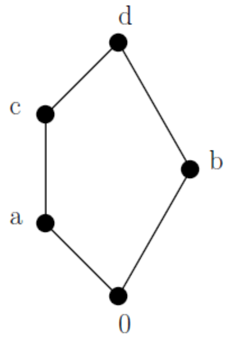

Figure 5. Example of f-semi prime but not a f-prime.

Then $(\mathbb{Q}, \leq)$ is a poset. Consider a mapping f from $\mathbb{Q}$ into set of ideals of $\mathbb{Q}$ such that $f(0) = \{0\}$, $f(a) = \{0, a\}$, $f(b) = \{0, b\}$, $f(c) = \{0, a, c\}$ and $f(d) = \{0, a, b, c, d\}$. Then $f \in \mathbb{F}(\mathbb{Q})$. Here $H = \{0\}$ is a f-semi prime ideal of $\mathbb{Q}$, not a f-prime as $(f(a)^*, f(b)^*)^\ell \subseteq H$ with $a \notin H$ and $b \notin H$.

Theorem 12. *The intersection of any non-empty family of f-prime ideals of $\mathbb{Q}$ is a f semi-prime ideal of $\mathbb{Q}$ for $f \in \mathbb{F}(\mathbb{Q})$.*

Proof. Let $H = \cap K_i$, where K_i 's are f-prime ideals of $\mathbb{Q}$ with $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq H$ for different proper ideals $f(\eta_1), f(\eta_2), f(\eta_3)$ of $\mathbb{Q}$. Then $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq K_i$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq K_i$ for all i. Since each K_i is a f semi-prime ideal of $\mathbb{Q}$, we get $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq K_i$ for all i which implies $(f(\eta_1)^*, (f(\eta_2)^*, f(\eta_3)^*)^u)^\ell \subseteq \cap K_i = H$. As the intersection of ideals is again an ideal of $\mathbb{Q}$, we have H is an ideal of $\mathbb{Q}$. So H is a f-semi-prime ideal of $\mathbb{Q}$. ∎

Definition. An ideal $H(\neq \mathbb{Q})$ is called irreducible if for any ideals H_1 and H_2 of $\mathbb{Q}$, $H = H_1 \cap H_2$ implies $H_1 = H$ or $H_2 = H$.

The following theorem gives the relation between the irreducible ideals and f-prime ideals of $\mathbb{Q}$.

Theorem 13. *Every f-prime ideal of $\mathbb{Q}$ is an irreducible ideal of $\mathbb{Q}$.*

Proof. Let H be a f prime ideal of $\mathbb{Q}$ and H_1, H_2 be ideals of $\mathbb{Q}$ with $H = H_1 \cap H_2$. If there exists $q_1 \in H_1 \backslash H$ and $q_2 \in H_2 \backslash H$, then $(f(q_1)^*, f(q_2)^*)^\ell \subseteq (q_1, q_2)^\ell \subseteq H_1 \cap H_2 \subseteq H$. Since H is a f-prime ideal of $\mathbb{Q}$, we have either $q_1 \in H$ or $q_2 \in H$, a contradiction. ∎

Remark 14. In common parlance, the converse of the preceding statement is not correct. In Example 11, let $H = \{0, a\}$ is a irreducible ideal of $\mathbb{Q}$, but it is not a f-prime ideal of $\mathbb{Q}$ as for the ideals $f(c)$ and $f(b)$, we have $(f(c)^*, f(b)^*)^\ell \subseteq H$, but $c \notin H$ and $b \notin H$.

4. f-SEMIPRIMENESS IN POSETS

In this section, we prove some properties and characterizations of f-prime ideals and f-semi-prime ideals in posets.

Reviewing [3], for a subset K and a semi-ideal J of $\mathbb{Q}$, we initiated

$$\langle K, J\rangle = \left\{t \in \mathbb{Q} : (a,t)^{\ell} \subseteq J \text{ for all a } \in K\right\} = \bigcap_{a\in K} \langle a, J\rangle.$$

We write $\langle s, J\rangle$ instead of $\langle \{s\}, J\rangle$ while $K = \{s\}$. It is evident $K \subseteq \langle\langle K, J\rangle, J\rangle$ and $t \in \langle\langle t, J\rangle, J\rangle$ for a semi-ideal J of $\mathbb{Q}$ for all $t \in \mathbb{Q}$. Furthermore, if $K \subseteq C$, then $\langle C, J\rangle \subseteq \langle K, J\rangle$ [2]. For all subset Q_1 of $\mathbb{Q}$ and a semi-ideal I_1 of $\mathbb{Q}$, it is easy to verify that $\langle\langle\langle Q_1, I_1\rangle, I_1\rangle, I_1\rangle = \langle Q_1, I_1\rangle$.

Definition. Let I be a semi-ideal of $\mathbb{Q}$. Then I satisfies $(*)$ condition if whenever $(A, B)^{\ell} \subseteq I$, then $A \subseteq \langle B, I\rangle$ for any subsets A and B of $\mathbb{Q}$.

Remark 15. In Example 1, let $A = \{0, r, s, t\}, B = \{0, s\}$ and $I = \{0, r, u\}$. Then $(A, B)^{\ell} \subseteq I$, but $A \not\subseteq \langle B, I\rangle = \{0, r, u\}$. So there exists a semi-ideal I of $\mathbb{Q}$ which is not satisfies $(*)$ condition.

Theorem 16. *Let $f \in \mathbb{F}(\mathbb{Q})$ and H be a f-semi-prime ideal of $\mathbb{Q}$ with $(*)$ condition. Then the following statement hold for $\eta_1, \eta_2, \eta_3 \in \mathbb{Q}$.*

(i) $(f(\eta_1)^*,\ f(\eta_2)^*)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$ *if and only if* $(f(\eta_3)^*, f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq H$.

(ii) $(f(\eta_3)^*,\ (f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell} \subseteq H$ *if and only if* $((f(\eta_1)^*,\ f(\eta_2)^*)^u)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$.

(iii) $\langle f(\eta_1), H\rangle = \mathbb{Q}$ *if and only if* $f(\eta_1) \subseteq H$.

Proof. (i) Let $(f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$ and $z \in (f(\eta_3)^*, f(\eta_1)^*, f(\eta_2)^*)^{\ell}$. Then $z \in (f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$ and $z \leq \eta_3$ as $\eta_3 \in f(\eta_3)^*$ which imply $z \in (z, f(\eta_3)^*)^{\ell} \subseteq H$. So $(f(\eta_3)^*, f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq H$.

Conversely, let $(f(\eta_3)^*, f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq H$ and $z \in (f(\eta_1)^*, f(\eta_2)^*)^{\ell}$. Then $z \in \langle f(\eta_3)^*, H\rangle$ as $(f(\eta_1)^*, f(\eta_2)^*)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$.

(ii) Suppose $(f(\eta_3)^*, (f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell} \subseteq H$ and let $z \in ((f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell}$. Then $(f(\eta_3)^*, z)^{\ell} \subseteq (f(\eta_3)^*,\ (f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell} \subseteq H$ which implies $z \in \langle f(\eta_3)^*, H\rangle$.

Conversely, if $((f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell} \subseteq \langle f(\eta_3)^*, H\rangle$, then $f(\eta_1)^* \subseteq \langle f(\eta_3)^*, H\rangle$ and $f(\eta_2)^* \subseteq \langle f(\eta_3)^*, H\rangle$ which imply $(f(\eta_1)^*, f(\eta_3)^*)^{\ell} \subseteq H$ and $(f(\eta_2)^*, f(\eta_3)^*)^{\ell} \subseteq H$. Since H is f-semi-prime ideal, we have $(f(\eta_3)^*, (f(\eta_1)^*, f(\eta_2)^*)^u)^{\ell} \subseteq H$.

(iii) Let $f(\eta_1) \subseteq H$. Then for all $q_1 \in f(\eta_1)$, we have $\langle q_1, H\rangle = \mathbb{Q}$, so $\langle f(\eta_1)^*, H\rangle = \bigcap_{q_1 \in f(\eta_1)} \langle q_1, H\rangle = \mathbb{Q}$.

Conversely, if $q_1 \in f(\eta_1)$, then $(r, q_1)^{\ell} \subseteq H$ for all $r \in \mathbb{Q}$ as $\langle f(\eta_1), H\rangle = \mathbb{Q}$ which gives $q_1 \in (q_1)^{\ell} \subseteq H$. So, $f(\eta_1) \subseteq H$. ∎

As immediate consequence of Theorem 16 is the below corollary.

Corollary 17. *For $a, \eta_1 \in \mathbb{Q}$ and $f \in \mathbb{F}(\mathbb{Q})$, we have $\langle a, f(\eta_1)\rangle = \mathbb{Q}$ if and only if $a \in f(\eta_1)$.*

Remark 18. For $a \in \mathbb{Q}$ and an ideal H of $\mathbb{Q}$, we have $\langle a, H\rangle$ is a semi ideal of $\mathbb{Q}$, but not necessary to be an ideal of $\mathbb{Q}$. In the Example 11, for an ideal $H = \{0, a\}$, we have $\langle c, H\rangle$ is not ideal as $((a, b)^u)^\ell = (d)^\ell = \{0, a, b, c, d\} \nsubseteq \langle c, H\rangle$.

Theorem 19. *Let H and $f(\eta_1)$ be ideals of $\mathbb{Q}$ for $\eta_1 \in \mathbb{Q}$ and $f \in \mathbb{F}(\mathbb{Q})$. If H is f-semi-prime with $(*)$ condition, then $\langle f(\eta_1), H\rangle$ is an ideal of $\mathbb{Q}$.*

Proof. Let $t_1, t_2 \in \langle f(\eta_1)^*, H\rangle$. Then $(f(t_1)^*, f(\eta_1)^*)^\ell \subseteq (t_1, f(\eta_1)^*)^\ell \subseteq H$ and $(f(t_2)^*, f(\eta_1)^*)^\ell \subseteq (t_2, f(\eta_1)^*)^\ell \subseteq H$. Since H is a f-semi-prime ideal of $\mathbb{Q}$, we have $(f(\eta_1)^*, (f(t_1)^*, f(t_2)^*)^u)^\ell \subseteq H$. By Theorem 16(ii), we have $((t_1, t_2)^u)^\ell \subseteq ((f(t_1)^*, f(t_2)^*)^u)^\ell \subseteq \langle f(\eta_1)^*, H\rangle$. So $\langle f(\eta_1)^*, H\rangle$ is an ideal of $\mathbb{Q}$. ∎

The following theorem is the characterization of f-semi-primeness in terms of $\langle f(\eta_1), H\rangle$ for an ideal H of $\mathbb{Q}$ and $\eta_1 \in \mathbb{Q}$.

Theorem 20. *Let H be an ideal of $\mathbb{Q}$ with $(*)$ condition. Then H is a f-semiprime ideal of $\mathbb{Q}$ if and only if $\langle f(\eta_1)^*, H\rangle$ is a f-semi-prime ideal of $\mathbb{Q}$ for $\eta_1 \in \mathbb{Q}$.*

Proof. Let I be a f-semi-prime ideal of $\mathbb{Q}$.

Case (i). If $f(\eta_1)^* \subseteq H$, then by Theorem 16(iii), we have $\langle f(\eta_1)^*, H\rangle = \mathbb{Q}$, so $\langle f(\eta_1)^*, H\rangle$ is a f-semi-prime ideal of $\mathbb{Q}$.

Case (ii). Let $f(\eta_1)^* \nsubseteq H$ and $f(\eta_2), f(\eta_3)$ and $f(\eta_4)$ be different proper ideals of $\mathbb{Q}$ for $\eta_2, \eta_3, \eta_4 \in \mathbb{Q}$ such that $(f(\eta_3)^*, f(\eta_2)^*)^\ell \subseteq \langle f(\eta_1)^*, H\rangle$ and $(f(\eta_3)^*, f(\eta_4)^*)^\ell \subseteq \langle f(\eta_1)^*, H\rangle$. Then $(f(\eta_1)^*, f(\eta_3)^*, f(\eta_4)^*)^\ell \subseteq H$ and by Theorem 16(i), $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq \langle f(\eta_2)^*, H\rangle$ and $(f(\eta_1)^*, f(\eta_3)^*)^\ell \subseteq \langle f(\eta_4)^*, H\rangle$.

Let $z \in (f(\eta_1)^*, f(\eta_3)^*, (f(\eta_2)^*, f(\eta_4)^*)^u)^\ell$. Then $z \in (f(\eta_1)^*, f(\eta_3)^*)^\ell$ and $z \in ((f(\eta_2)^*, f(\eta_4)^*)^u)^\ell$ which imply $(f(\eta_2)^*, f(z)^*)^\ell \subseteq (f(\eta_2)^*, z)^\ell \subseteq (f(\eta_1)^*, f(\eta_2)^*, f(\eta_3)^*)^\ell \subseteq H$ and $(f(\eta_4)^*, f(z)^*)^\ell \subseteq (f(\eta_4)^*, z)^\ell \subseteq (f(\eta_1)^*, f(\eta_2)^*, f(\eta_3)^*)^\ell \subseteq H$. Hence $f(\eta_2)^*, f(\eta_4)^* \subseteq \langle f(z)^*, H\rangle$. By Theorem 19, $\langle f(z)^*, H\rangle$ is an ideal of $\mathbb{Q}$ and $z \in ((f(\eta_2)^*, f(\eta_4)^*)^u)^\ell \subseteq \langle f(z)^*, H\rangle = \bigcap_{t\in((z)^\ell)^*}\langle t, H\rangle$. So $z \in H$. Thus $(f(\eta_1)^*, f(\eta_3)^*, (f(\eta_2)^*, f(\eta_4)^*)^u)^\ell \subseteq H$ and $(f(\eta_3)^*, (f(\eta_2)^*, f(\eta_4)^*)^u)^\ell \subseteq \langle f(\eta_1)^*, H\rangle$.

Conversely, let $\langle f(\eta_1)^*, H\rangle$ be a f-semi-prime ideal of $\mathbb{Q}$ for any ideal $f(\eta_1)$ of $\mathbb{Q}$. Suppose $f(\eta_2), f(\eta_3)$ and $f(\eta_4)$ are different proper ideals of $\mathbb{Q}$ such that $(f(\eta_2)^*, f(\eta_3)^*)^\ell \subseteq H$ and $(f(\eta_2)^*, f(\eta_4)^*)^\ell \subseteq H$. Then $(f(\eta_2)^*, f(\eta_3)^*)^\ell \subseteq \langle f(\eta_2)^*, H\rangle$ and $(f(\eta_2)^*, f(\eta_4)^*)^\ell \subseteq \langle f(\eta_2)^*, H\rangle$. Since $\langle f(\eta_2)^*, H\rangle$ is f-semi-prime, we have $(f(\eta_2)^*, (f(\eta_3)^*, f(\eta_4)^*)^u)^\ell \subseteq \langle f(\eta_2)^*, H\rangle$.

Let $t \in (f(\eta_2)^*, (f(\eta_3)^*, f(\eta_4)^*)^u)^\ell$. Then $(f(\eta_2)^*, t)^\ell \subseteq H$. Since $t \leq s$ for all $s \in f(\eta_2)^*$, we have $t \in H$. Hence $(f(\eta_2)^*, (f(\eta_3)^*, f(\eta_4)^*)^u)^\ell \subseteq H$. ■

As immediate consequence of Theorem 20, we have the following corollaries.

Corollary 21 ([11], Theorem 15)**.** *Let H be an ideal of $\mathbb{Q}$. Then H is semi-prime if and only if $\langle q, H\rangle$ is a semi-prime ideal of $\mathbb{Q}$ for all $q \in \mathbb{Q}$.*

Corollary 22. *Let H be an ideal of $\mathbb{Q}$. Then H is a semi-prime ideal of $\mathbb{Q}$ if and only if $\langle R, H\rangle$ is a semi-prime ideal of $\mathbb{Q}$ for all $R \subseteq \mathbb{Q}$.*

Proof. Let H be a semi-prime ideal of $\mathbb{Q}$ and $R \subseteq H$. Then by Corollary 21, we have $\langle a, H\rangle$ is a semi-prime ideal of $\mathbb{Q}$ and $\langle R, H\rangle = \bigcap_{a\in R}\langle a, H\rangle$. Again by intersection of semi-prime ideals is a semi-prime ideal, we have $\langle R, H\rangle$ is a semi-prime ideal of $\mathbb{Q}$. ■

Theorem 23. *Let H be a maximal ideal of $\mathbb{Q}$ with $(*)$ condition. Then H is f-prime if and only if H is f-semi-prime.*

Proof. Let H be a maximal and f-semi-prime ideal of $\mathbb{Q}$. Suppose that $f(\eta_1)$ and $f(\eta_2)$ are different proper ideals of $\mathbb{Q}$ such that $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$. Then $f(\eta_1)^* \subseteq \langle f(\eta_2)^*, H\rangle$ and by Theorem 20, $\langle f(\eta_2)^*, H\rangle$ is a f-semi-prime ideal of $\mathbb{Q}$. Since $H \subseteq \langle f(\eta_2)^*, H\rangle$ and by maximality of H, $\langle f(\eta_2)^*, H\rangle = \mathbb{Q}$. By Theorem 16(iii), we have $f(\eta_2) \subseteq H$. ■

Remark 24 ([11], Theorem 16 and Corollary 17)**.** For a maximal ideal H of $\mathbb{Q}$, we have H is semi prime if and only if H is prime.

Theorem 25. *Let $f(r)$ be a f-prime ideal of $\mathbb{Q}$ for some $r \in \mathbb{Q}$. Then $\langle f(\eta_1)^*, f(r)\rangle = f(r)$ for all ideal $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$.*

Proof. Suppose $f(r)$ is a f-prime and $f(\eta_1)$ is an ideal of $\mathbb{Q}$ for some $r, \eta_1 \in \mathbb{Q}$ such that $f(\eta_1) \nsubseteq f(r)$. Clearly $f(r) \subseteq \langle f(\eta_1)^*, f(r)\rangle$ is always true. Let $z \in \langle f(\eta_1)^*, f(r)\rangle$. Then $(f(z)^*, f(\eta_1)^*)^\ell \subseteq (z, f(\eta_1)^*)^\ell \subseteq f(r)$. Since $f(r)$ is f-prime and $f(\eta_1) \nsubseteq f(r)$, we have $z \in f(r)$. ■

The next Theorem gives some equivalent conditions for f-prime ideals.

Theorem 26. *Let $f(r)$ be an ideal of $\mathbb{Q}$ with $(*)$ condition. Then the following are equivalent.*

(i) *$f(r)$ is a f-prime ideal of $\mathbb{Q}$,*

(ii) *$\langle f(\eta_1)^*, f(r)\rangle = f(r)$ for any ideal $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$,*

(iii) *$f(r)$ is a prime ideal of $\mathbb{Q}$,*

(iv) *$\langle x, f(r)\rangle$ is a f-prime ideal of $\mathbb{Q}$ for all $x \in \mathbb{Q}\backslash f(r)$.*

Proof. (i) $\Rightarrow$ (ii) If $f(r)$ is f-prime, then by Theorem 25, we have $\langle f(\eta_1)^*, f(r)\rangle = f(r)$ for all ideal $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$.

(ii) $\Rightarrow$ (iii) Let $\langle f(\eta_1)^*, f(r)\rangle = f(r)$ for all ideals $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$ and $(x,y)^\ell \subseteq f(r)$ for $x, y \in \mathbb{Q}$. If $y \notin f(r)$, then $(x, f(y)^*)^\ell \subseteq (x,y)^\ell \subseteq f(r)$ which implies $x \in \langle f(y)^*, f(r)\rangle = f(r)$.

(iii) $\Rightarrow$ (iv) Let $f(r)$ be a prime ideal of $\mathbb{Q}$ and $z \in \langle x, f(r)\rangle$ for $x \in \mathbb{Q}\backslash f(r)$. Then $(x,z)^\ell \subseteq f(r)$. Since $f(r)$ is prime and $x \notin f(r)$, we have $z \in f(r)$. So $\langle x, f(r)\rangle \subseteq f(r)$ and clearly $f(r) \subseteq \langle x, f(r)\rangle$. Hence $\langle x, f(r)\rangle = f(r)$ for all $x \in \mathbb{Q}\backslash f(r)$.

(iv) $\Rightarrow$ (i) Let $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq f(r)$ for different proper ideals $f(\eta_1)$ and $f(\eta_2)$ of $\mathbb{Q}$. If $f(\eta_1) \nsubseteq f(r)$, then there exists $t \in f(\eta_1)\backslash f(r)$. Since $f(r)$ has $(*)$ condition, we have $f(\eta_2)^* \subseteq \langle f(\eta_1)^*, f(r)\rangle = \bigcap_{a\in f(\eta_1)^*}\langle a, f(r)\rangle \subseteq \langle t, f(r)\rangle = f(r)$ and hence $f(r)$ is a f-prime ideal of $\mathbb{Q}$. ■

Corollary 27. *Let $f(r)$ be a semi-ideal of $\mathbb{Q}$. Then f(r) is prime if and only if $\langle x, f(r)\rangle = f(r)$ for all $x \in \mathbb{Q}\backslash f(r)$.*

Corollary 28. *Let $f(r)$ be an ideal of $\mathbb{Q}$. Then $f(r)$ is prime if and only if $\langle x, f(r)\rangle = f(r)$ for all $x \in \mathbb{Q}\backslash f(r)$.*

Corollary 29. *Let $f(r)$ be an ideal of $\mathbb{Q}$ with $(*)$ condition. Then $f(r)$ is f-prime if and only if $f(r)$ is prime.*

Corollary 30. *Let $f(r)$ be an ideal of $\mathbb{Q}$. If $f(r)$ is prime, then $\langle x, f(r)\rangle$ is a prime ideal of $\mathbb{Q}$ for all $x \in \mathbb{Q}\backslash f(r)$*

The classification of f-primeness is obtained from the preceding theorem in terms of $\langle f(\eta_1)^*, f(r)\rangle$ for ideals $f(\eta_1), f(r)$ of $\mathbb{Q}$.

Theorem 31. *Let $f(r)$ be an ideal of $\mathbb{Q}$ with $(*)$ condition for $r \in \mathbb{Q}$. If $f(r)$ is f-prime, then $\langle f(\eta_1)^*, I\rangle$ is a f-prime ideal of $\mathbb{Q}$ for ideal $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$.*

Proof. Let $f(r)$ be a f-prime ideal of $\mathbb{Q}$. Then by Theorem 26, we have $\langle f(\eta_1)^*, f(r)\rangle = f(r)$ for ideal $f(\eta_1)$ of $\mathbb{Q}$ not contained in $f(r)$ and hence $\langle f(\eta_1)^*, f(r)\rangle$ is a f-prime ideal of $\mathbb{Q}$. ■

Corollary 32. *Let $f(r)$ be an ideal of $\mathbb{Q}$ with $(*)$ condition for $r \in \mathbb{Q}$. If $f(r)$ is f-prime, then $\langle x, f(r)\rangle$ is a f-prime ideal of $\mathbb{Q}$ for all $x \in \mathbb{Q}\backslash f(r)$.*

The following example shows that the converse of the Theorem 31 is not true in general.

Example 33. In Example 11, if we take $I = \{0\}$ and $f(a) = \{0, a\}$, then $\langle f(a)^*, I\rangle = \{0, b\}$ is a f-prime ideal of $\mathbb{Q}$, but I is not a f-prime ideal of $\mathbb{Q}$ for the ideals $f(b) = \{0, b\}, f(c) = \{0, a, c\}$ of $\mathbb{Q}, (f(b)^*, f(c)^*)^\ell \subseteq I$ with $b \nsubseteq I$ and $c \nsubseteq I$.

5. Properties of the set C_H

Definition. For an ideal H of $\mathbb{Q}$, we indicate the set $C_H = \{w \in \mathbb{Q} : \langle w, H\rangle = H\}$.

We developed the several characteristics of C_H and its correlation with H in the following results.

Lemma 34. *Let I be a f-semiprime ideal of $\mathbb{Q}$. Then $\langle f(\eta_1)^*, I\rangle \cap C_I = \emptyset$ for all ideals $f(\eta_1)$ of $\mathbb{Q}$ not contained in I.*

Following [19], a subset $B(\neq \emptyset)$ of $\mathbb{Q}$ is termed as semi-filter if $s \in B$ and $s \leqslant q$, then $q \in B$. Also B is referred as filter if $s, d \in B$ implies $(s, d)^{\ell u} \subseteq B$ [9].

Theorem 35. *Let I be an ideal of $\mathbb{Q}$. Then C_I is a filter of $\mathbb{Q}$.*

Lemma 36. *Let I be a proper ideal of $\mathbb{Q}$. Then $I \cap C_I = \emptyset$.*

The following theorem characterizes f-prime ideals in a poset.

Theorem 37. *Let H be a proper ideal of $\mathbb{Q}$ with $(*)$ condition. Then H is f-prime if and only if $H \cup C_H = \mathbb{Q}$.*

Proof. Suppose H is a f-prime ideal of $\mathbb{Q}$ and let $x \notin C_H$ for $x \in \mathbb{Q}$. Then $\langle x, H\rangle \neq H$ which implies $y \in \langle x, H\rangle$ with $y \notin H$ and $(f(x)^*, f(y)^*)^\ell \subseteq (x, y)^\ell \in H$. Since H is f-prime ideal and $y \notin H$ which imply $x \in H$.

Conversely, let $H \cup C_H = \mathbb{Q}$ and $f(\eta_1), f(\eta_2)$ be different proper ideals of $\mathbb{Q}$ with $(f(\eta_1)^*, f(\eta_2)^*)^\ell \subseteq H$ for $\eta_1, \eta_2 \in \mathbb{Q}$. If $\eta_1 \notin H$, then $f(\eta_2)^* \subseteq \langle f(\eta_1)^*, H\rangle$ and there exists $a \in f(\eta_1)\backslash H$ with $\langle a, H\rangle = H$ which imply $\eta_2 \in f(\eta_2)^* \subseteq \bigcap_{t\in f(\eta_1)^*} \langle t, H\rangle \subseteq \langle a, H\rangle = H$. ∎

Corollary 38. *Let H be a proper ideal of $\mathbb{Q}$. Then H is prime if and only if $H \cup C_H = \mathbb{Q}$.*

6. Conclusion

We investigated the ideas of f-prime ideals and f-semi-prime ideals of posets in this work, as well as the different features of f-primeness and f-semi primeness in posets. Characterizations of f-semi-prime ideals in posets are derived, in furthermore categorizations of a f-semi-prime ideal as f-prime. We established some fundamental theorems in f-primeness and obtained equivalent criteria for a semi-ideal of $\mathbb{Q}$ to be a f-prime semi-ideals of $\mathbb{Q}$. In addition, we discussed the requirements for an ideal to be a f-prime ideal of $\mathbb{Q}$. These findings may be extended to 0-distributive posets, lattices, near lattices, semilattices, and 0-distributive near lattices using the technique presented in this paper.

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