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ERRATUM TO: "APPLICATION OF (m, n)- Γ-HYPERIDEALS IN CHARACTERIZATION OF LA- Γ-SEMIHYPERGROUPS" BY ABUL BASAR

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Abstract

This note is written to show that the definition of the LA-Γ-hypersemigroup and the definition of the ordered LA-Γ-hypersemigroup in [2] should be corrected and that it is not enough to replace the "Γ" of the ordered LA-Γ-semigroup by "$\circ\,\Gamma\circ$" to pass from an ordered LA-Γ-semigroup to an ordered LA-Γ-hypersemigroup. The definition of the (m, n)-Γ-hyperideal and the strange symbols used throughout the paper have no sense as well.

Keywords: LA-Γ-semihypergroup, ordered LA-Γ-semihypergroup, (m, n)-Γ-hyperideal.

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There is nothing correct in this paper. The paper is based on definitions without any sense. In spite of its title, the paper was intended to be on ordered LA-Γ-semihypergroups. First of all the definition of the LA-semihypergroup given in the introduction should be corrected, the existing definition defines the semihypergroups and not the LA-semihypergroups. Then the definition of an ordered LA-Γ-semihypergroup is given as follows: "An LA-semihypergroup $(S, \circ)$ together with a partial order "$\leq$" on S that is compatible with the LA-semihypergroup operation such that $x \leq y$ implies $z \circ \alpha \circ x \leq z \circ \beta \circ y$ and $x \circ \alpha \circ z \leq y \circ \beta \circ z$ for all $x, y, z \in S$ and all $\alpha, \beta \in \Gamma$ is called an LA-Γ-semihypergroup" (see p. 137, l. 1–5). There is no Γ in the definition. Except of the fact that the definition of the LA-semihypergroup and the LA-Γ-hypersemigroup should be corrected, expressions of the form $z \circ \alpha \circ x$, $z \circ \beta \circ y$ do not have any sense, as $\circ$ is an "operation" (the so called hyperoperation as it assigns to each couple of elements (a, b) a nonempty subset of S) between elements of S and not between elements of S and Γ and the $\leq$ is the order of S (i.e., relation between elements of S), so we can never write the $\leq$ between the $z \circ \alpha \circ x$ and $z \circ \beta \circ y$, where $z \circ \alpha \circ x$ and $z \circ \beta \circ y$ (after correction) should be sets and not elements. That is, we can never write $x \leq y$ implies $z \circ \alpha \circ x \leq z \circ \beta \circ y$ and $x \circ \alpha \circ z \leq y \circ \beta \circ z$ for all $x, y, z \in S$ and all $\alpha, \beta \in \Gamma$; first of all because we cannot write $x \circ \alpha \circ y$, where $x, y \in S$ and $\alpha \in \Gamma$ and secondly because the order on S cannot be an order on the set of all nonempty subsets of S. And certainly the "left invertible law" (p. 137, l. 5–6) defined as $(x \circ \gamma \circ y) \circ \beta \circ z = (z \circ \gamma \circ y) \circ \beta \circ x$ cannot have any meaning as well.

This is from p. 137, l. 7–11: "For subsets A, B of an LA-Γ-semihypergroup S, the product set $A \circ B$ of the pair (A, B) relative to S is defined as $A \circ \Gamma \circ B = \{a \circ \gamma \circ b \mid a \in A, b \in B, \gamma \in \Gamma\}$, and for $A \subseteq S$, the product set $A \circ \Gamma \circ A$ relative to S is defined by $A^2 = A \circ A = A \circ \Gamma \circ A$. Note that A^0 acts as an identity operator. That is, $A^0 \circ \Gamma \circ S = S = S \circ \Gamma \circ A^0$."

Can $A \circ B$ be equal to $A \circ \Gamma \circ B$ and $A^2 = A \circ A = A \circ \Gamma \circ A$?

But anyway, the $A \circ \Gamma \circ B$ has been defined as the set $\{a \circ \gamma \circ b \mid a \in A, b \in B, \gamma \in \Gamma\}$. We are not in a Γ-semigroup to write $A\Gamma B = \{a\gamma b \mid a, b \in A, \gamma \in \Gamma\}$ [3]; but even in a Γ-semigroup we can never write $A \cdot \Gamma \cdot B$ (that is certainly wrong). If we correct the $A \circ \Gamma \circ B$ and write $A\Gamma B$ instead (after giving the correct definition of the LA-Γ-hypersemigroup of course), this cannot be a set of sets, that is, something like $\{a \circ \gamma \circ b \mid a \in A, b \in B, \gamma \in \Gamma\}$ (as the author claims), but it should be a union of sets.

There are 6 properties on the same page 137, l. 14–19, that are copied from similar results on ordered semigroups, putting "$\circ\ \Gamma\circ$" instead of "$\cdot$", for which the author says "we easily have the following (properties)" giving the impression they due to him. These properties come from the ordered semigroups, in four of them the hyperoperation does not play any role, two of them without sense, no reference to ordered semigroups is given.

The definition of an (m, n)-Γ-hyperideal given in the paper (Definition 2.1) is as follows: "Suppose $(S, \Gamma, \circ, \leq)$ is an ordered LA-Γ-semihypergroup and m, n nonnegative integers. An LA-sub-Γ-semihypergroup A is called an (m, n)-Γ-hyperideal of S if (1) $A^m \circ \Gamma \circ S \circ \Gamma \circ A^n \subseteq A$ and (2) for any $a \in A$ and $s \in S$, $s \leq a$ implies $s \in A$". But what the $A^m \circ \Gamma \circ S \circ \Gamma \circ A^n \subseteq A$ means? Let us consider just an hypersemigroup, and write $A^n = A \circ A \circ \cdots \circ A$. For $n = 2$, A^2 means $A \circ A$ and this is the set $\bigcup\limits_{a \in A, b \in A} a \circ b$.

For $n = 3$, $A^3 = (A \circ A) \circ A = \bigcup\limits_{x \in A \circ A, y \in A} x \circ y = \bigcup\limits_{x \in \bigcup\limits_{a \in A, b \in A} a \circ b, y \in A} x \circ y.$

For $n = 4$, $A^4 = (A \circ A) \circ (A \circ A) = \bigcup\limits_{x \in A \circ A, y \in A \circ A} x \circ y = \bigcup\limits_{x \in \bigcup\limits_{a \in A, b \in A} a \circ b, y \in \bigcup\limits_{c \in A, d \in A} c \circ d} x \circ y.$

As we see, even for $n = 3$ the case is complicated. Is it possible to continue this way, define the A^n and work on it? This is no possible even for an hypersemigroup (that is for a Γ-hypersemigroup for which the set Γ is a singleton). As an equivalent definition of the (m,n)-Γ-hyperideal of an LA-Γ-hypersemigroup, the following is written: "Equivalently: an ordered LA-Γ-semihypergroup A of S is called (m,n)-Γ-hyperideal of S if $(A^m \circ \Gamma \circ S \circ \Gamma \circ A^n] \subseteq A$". It is not written in the right way, but the reader understands what the author means. Throughout the paper this second "equivalent" condition of the (m,n)-Γ-hyperideal has been used and the analogous in all results in [1]. But the $(A^m \circ \Gamma \circ S \circ \Gamma \circ A^n] \subseteq A$ cannot be equivalent relation. If it were true (after giving the correct definition of course that cannot contain $\circ$ between sets and Γ), that meant that a bi-ideal A of an ordered semigroup S can be equivalently defined as $(ASA] \subseteq A$ that is certainly wrong. For every bi-ideal A of an ordered semigroup S, we have $(ASA] \subseteq A$ but the converse statement does not hold in general. Here we should mention that, in the simple case of an hypergroupoid S one could say "we write $x \circ (y \circ z) = (x \circ y) \circ z$ if the condition $(*)$ defined by $\bigcup\limits_{u \in y \circ z} x \circ u = \bigcup\limits_{v \in x \circ y} v \circ z$ holds", but we never use the condition $(*)$ inside the text; and expressions of the form $x \circ (y \circ z)$ are just symbols that show what we mean when we write $x \circ (y \circ z) = (x \circ y) \circ z$. If we use these symbols in the proofs and develop a theory based on these symbols without any explanation to clarify our investigation, then this is certainly wrong. In any more general case of an hypersemigroup or LA-hypersemigroup (in Γ-hypersemigroup, LA-Γ-hypersemigroup, for example) the same argument holds.

Throughout the paper there are strange symbols like, for example, $((R^m \circ \Gamma \circ R^n \circ \Gamma \circ S \circ \Gamma \circ S) \circ \Gamma \circ (L^m \circ \Gamma \circ L^n \circ \Gamma \circ S \circ \Gamma \circ S)]$ (p. 138, l.-1); $((s^{mn} \circ \Gamma \circ (S \circ \Gamma \circ ((s^{mm} \circ \Gamma \circ S^m) \circ \Gamma \circ s^{mn})) \circ \Gamma \circ S^n) \circ \Gamma \circ s^{nn}]$ (p. 143, l.-11) without any sense, the last even worse than the previous one, because the operation is between sets and elements here. What is the $(S \circ S \circ \Gamma \circ M^2]$ in the proof of Theorem 3.2 ? (p. 141, l. -2). We are not in a semigroup or in a Γ-semigroup in which such expressions have meaning.

In what follows, the aim is to show that is not enough to pass from an ordered LA-Γ-semigroup to an ordered LA-Γ-hypersemigroup by replacing the "Γ" of the LA-Γ-semigroup by "$\circ\,\Gamma\circ$".

The paper in [2] is the paper in [1] with the only difference that the "Γ" in [1] has been replaced by "$\circ\,\Gamma\circ$" in [2].

In fact, the paper in [2] consists of the Example 3.1, Lemma 3.1, Theorem 3.1, Theorem 3.2, Theorem 3.3, Lemma 3.2 and Theorem 3.4; and

Example 3.1 in [2] is the Example 2.1 in [1] (the order is missing in [1]) in which the letters x, y, z, w, e have been replaced by a, b, c, d, e. Again here, the expression $a \circ \gamma \circ b$ is without sense. In addition, this example is from ordered semigroups and does not belong to the author, so a reference was needed for Example 3.1.

Lemma 3.1 in [2] is the Lemma 2.2 in [1].

Theorem 3.1 in [2] is the Theorem 2.3 in [1].
Theorem 3.2 in [2] is the Theorem 2.4 in [1].
Theorem 3.3 in [2] is the Theorem 2.5 in [1].
Lemma 3.2 in [2] is the Lemma 2.6 in [1].
Theorem 3.4 in [2] is the Theorem 2.7 in [1].

The paper in [1] is consisted by published results [4; p. 284–288] in which a ( ] has been casually, and a Γ has been added and contains serious mistakes; but this is out of the scope of the present note. For Lemma 2.2, Theorem 2.3, Theorem 2.4, Theorem 2.5, Lemma 2.6 and Theorem 2.7 in [1] the reader could see the Lemma 7, Theorem 6, Theorem 7, Theorem 8, Lemma 8 and Theorem 9 in [4], respectively.

All the results of the paper are on an "unitary LA-Γ-hypersemigroup" but there is no the definition of the "unitary" in the paper.

It should be mentioned here that many results on Γ-semigroups (ordered Γ-semigroups) can be obtained from semigroups (ordered semigroups) just putting a Gamma in the appropriate place (see [3]). Finally, 41 papers are cited in the References (most of them not related with the paper), while this paper is based only on the 1–2 papers mentioned above, and they should be indicated inside the paper; only in References is not enough. The introduction of the paper contains mistakes.

The author should first define the LA-Γ-hypersemigroup correctly. The correct definition of the Γ-hypersemigroup has been given in a paper submitted by the author of the present note to Turkish J. Math. If we have the correct definition of the Γ-hypersemigroup, then we have the correct definition of the LA-Γ-hypersemigroup as well. But, anyway any result on hypersemigroups, ordered hypersemigroups, Γ-hypersemigroups, ordered Γ-hypersemigroups, LA-semigroups, etc. comes from corresponding results on semigroups, we just have to introduce the right definitions on which the results are based. We never work directly on these structures, however, it could be instructive to show how independent proofs work.

References

[1] A. Basar, *A note on (m, n)-Γ-ideals of ordered LA-Γ-semigroups*, Konuralp J. Math. **7** (2019) 107–111.

[2] A. Basar, *Application of (m, n)-Γ-hyperideals in characterization of LA-Γ-semihypergroups*, Discuss. Math. Gen. Algebra Appl. **39** (2019) 135–147.

[3] N. Kehayopulu, *On ordered Γ-semigroups*, Sci. Math. Jpn. **71** (2010) 179–185.

[4] W. Khan, F. Yousafzai and M. Khan, *On generalized ideals of left almost semigroups*, Eur. J. Pure Appl. Math. **9** (2016) 277–291.

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