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INTRODUCING FULLY UP-SEMIGROUPS [1]

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Abstract

In this paper, we introduce some new classes of algebras related to UP-algebras and semigroups, called a left UP-semigroup, a right UP-semigroup, a fully UP-semigroup, a left-left UP-semigroup, a right-left UP-semigroup, a left-right UP-semigroup, a right-right UP-semigroup, a fully-left UP-semigroup, a fully-right UP-semigroup, a left-fully UP-semigroup, a right-fully UP-semigroup, a fully-fully UP-semigroup, and find their examples.

Keywords: semigroup, UP-algebra, fully UP-semigroup.

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1. Introduction and preliminaries

In the literature, several researchers introduced a new class of algebras related to logical algebras and semigroups such as: In 1993, Jun, Hong and Roh [4] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup. In 1998, Jun, Xin, and Roh [5, 6] renamed the BCI-semigroup as the IS-algebra and studied further properties of these algebras. In 2006, Kim [8] introduced the notion of KS-semigroups. In 2011, Ahn and Kim [1] introduced the notion of BE-semigroups. In 2015, Endam and Vilela [2] introduced the notion of JB-semigroups. In 2016, Sultana and Chaudhary [11] introduced the notion of BCH-semigroups. In 2018, Kareem and Hasan introduced and analyzed the concept of KU-semigroups in the recently published article [7]. It is known that UP-algebra is a generalization of KU-algebra [3]. Several authors also studied the algebraic structures with semigroups (see, for example: [1, 8–11]).

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In this paper, we introduce some new classes of algebras related to UP-algebras and semigroups, called a left UP-semigroup, a right UP-semigroup, a fully UP-semigroup, a left-left UP-semigroup, a right-left UP-semigroup, a left-right UP-semigroup, a right-right UP-semigroup, a fully-left UP-semigroup, a fully-right UP-semigroup, a left-fully UP-semigroup, a right-fully UP-semigroup, a fully-fully UP-semigroup, and find their examples.

Before we begin our study, we will introduce the definition of a UP-algebra.

Definition 1.1 [3]**.** An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where A is a nonempty set, $\cdot$ is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms: for any $x, y, z \in A$,

(UP-1) $(y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0$,

(UP-2) $0 \cdot x = x$,

(UP-3) $x \cdot 0 = 0$, and

(UP-4) $x \cdot y = 0$ and $y \cdot x = 0$ imply $x = y$.

Proposition 1.2. *In a UP-algebra* $A = (A, \cdot, 0)$*, the following assertions are valid* ((1.1)–(1.7), *see* [3], *Proposition* 1.7).

(1.1) $(\forall x \in A)(x \cdot x = 0)$,

(1.2) $(\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0)$,

(1.3) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0)$,

(1.4) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0)$,

(1.5) $(\forall x, y \in A)(x \cdot (y \cdot x) = 0)$,

(1.6) $(\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x)$,

(1.7) $(\forall x, y \in A)(x \cdot (y \cdot y) = 0)$,

(1.8) $(\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0)$,

(1.9) $(\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0)$,

(1.10) $(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0)$,

(1.11) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0)$,

(1.12) $(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0)$, *and*

(1.13) $(\forall a, x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0)$.

Proof. (1.8) By (UP-1), we have $(y \cdot z) \cdot ((a \cdot y) \cdot (a \cdot z)) = 0$. By (1.3), we have

$$(x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0.$$

(1.9) By (UP-1), we have $(x \cdot y) \cdot ((a \cdot x) \cdot (a \cdot y)) = 0$. By (1.4), we have

$$(((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0.$$

(1.10) Now,

$$\begin{aligned}
&((1.9)) && 0 = (((x \cdot 0) \cdot (x \cdot y)) \cdot z) \cdot ((0 \cdot y) \cdot z) \\
&((\text{UP-2}), (\text{UP-3})) && \quad = ((0 \cdot (x \cdot y)) \cdot z) \cdot (y \cdot z) \\
&((\text{UP-2})) && \quad = ((x \cdot y) \cdot z) \cdot (y \cdot z).
\end{aligned}$$

Hence, $((x \cdot y) \cdot z) \cdot (y \cdot z) = 0$.

(1.11) Assume that $x \cdot y = 0$. By (1.3), we have $(z \cdot x) \cdot (z \cdot y) = 0$. By (1.10) and (UP-2), we have

$$x \cdot (z \cdot y) = 0 \cdot (x \cdot (z \cdot y)) = ((z \cdot x) \cdot (z \cdot y)) \cdot (x \cdot (z \cdot y)) = 0.$$

Hence, $x \cdot (z \cdot y) = 0$.

(1.12) By (1.10), we have

$$((x \cdot y) \cdot z) \cdot (y \cdot z) = 0.$$

By (1.5), we have

$$(y \cdot z) \cdot (x \cdot (y \cdot z)) = 0.$$

It follows from (1.2) that $((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0$.

(1.13) By (1.5), we have $y \cdot (x \cdot y) = 0$ and $(x \cdot y) \cdot (a \cdot (x \cdot y)) = 0$. By (1.2), we have $y \cdot (a \cdot (x \cdot y)) = 0$. By (1.4), we have

$$((a \cdot (x \cdot y)) \cdot (a \cdot z)) \cdot (y \cdot (a \cdot z)) = 0.$$

By (UP-1), we have

$$((x \cdot y) \cdot z) \cdot ((a \cdot (x \cdot y)) \cdot (a \cdot z)) = 0.$$

It follows from (1.2) that $((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0$. ■

Let X be a universal set. Define two binary operations $\cdot$ and $*$ on the power set of X by putting, for all $A, B \in \mathcal{P}(X)$,

$$A \cdot B = A' \cap B, \tag{1.14}$$

$$A * B = A' \cup B \tag{1.15}$$

where A' means the complement of a subset A. Then $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type* 1 [3], Example 1.4, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type* 2 [3], Example 1.5.

Now, define four binary operations $\odot, \otimes, \boxdot$ and $\boxtimes$ on the power set of X by putting, for all $A, B \in \mathcal{P}(X)$,

$$A \odot B = X, \tag{1.16}$$

$$A \otimes B = \emptyset, \tag{1.17}$$

$$A \boxdot B = B, \tag{1.18}$$

$$A \boxtimes B = A. \tag{1.19}$$

Then $(\mathcal{P}(X), \odot), (\mathcal{P}(X), \otimes), (\mathcal{P}(X), \boxdot)$ and $(\mathcal{P}(X), \boxtimes)$ are semigroups which is determined by direct verification. Furthermore, we know that $(\mathcal{P}(X), \cap, X)$ and $(\mathcal{P}(X), \cup, \emptyset)$ are monoids.

Definition 1.3. Let A be a nonempty set, $\cdot$ and $*$ are binary operations on A, and 0 is a fixed element of A (i.e., a nullary operation). An algebra $A = (A, \cdot, *, 0)$ of type $(2, 2, 0)$ in which $(A, \cdot, 0)$ is a UP-algebra and $(A, *)$ is a semigroup is called

(1) a *left UP-semigroup* (in short, an *l-UP-semigroup*) if the operation "$*$" is left distributive over the operation "$\cdot$",

(2) a *right UP-semigroup* (in short, an *r-UP-semigroup*) if the operation "$*$" is right distributive over the operation "$\cdot$",

(3) a *fully UP-semigroup* (in short, an *f-UP-semigroup*) if the operation "$*$" is distributive (on both sides) over the operation "$\cdot$",

(4) a *left-left UP-semigroup* (in short, an *(l, l)-UP-semigroup*) if the operation "$\cdot$" is left distributive over the operation "$*$" and the operation "$*$" is left distributive over the operation "$\cdot$",

(5) a *right-left UP-semigroup* (in short, an *(r, l)-UP-semigroup*) if the operation "$\cdot$" is right distributive over the operation "$*$" and the operation "$*$" is left distributive over the operation "$\cdot$",

(6) a *left-right UP-semigroup* (in short, an *(l, r)-UP-semigroup*) if the operation "$\cdot$" is left distributive over the operation "$*$" and the operation "$*$" is right distributive over the operation "$\cdot$",

(7) a *right-right UP-semigroup* (in short, an *(r, r)-UP-semigroup*) if the operation "$\cdot$" is right distributive over the operation "$*$" and the operation "$*$" is right distributive over the operation "$\cdot$",

(8) a *fully-left UP-semigroup* (in short, an *(f, l)-UP-semigroup*) if the operation "$\cdot$" is distributive (on both sides) over the operation "$*$" and the operation "$*$" is left distributive over the operation "$\cdot$",

(9) a *fully-right UP-semigroup* (in short, an *(f, r)-UP-semigroup*) if the operation "$\cdot$" is distributive (on both sides) over the operation "$*$" and the operation "$*$" is right distributive over the operation "$\cdot$",

(10) a *left-fully UP-semigroup* (in short, an (l, f)-*UP-semigroup*) if the operation "$\cdot$" is left distributive over the operation "$*$" and the operation "$*$" is distributive (on both sides) over the operation "$\cdot$",

(11) a *right-fully UP-semigroup* (in short, an (r, f)-*UP-semigroup*) if the operation "$\cdot$" is right distributive over the operation "$*$" and the operation "$*$" is distributive (on both sides) over the operation "$\cdot$", and

(12) a *fully-fully UP-semigroup* (in short, an (f, f)-*UP-semigroup*) if the operation "$\cdot$" is distributive (on both sides) over the operation "$*$" and the operation "$*$" is distributive (on both sides) over the operation "$\cdot$".

In what follows, let A and B denote UP-algebras unless otherwise specified. The following proposition is very important for the study of UP-algebras.

The proof of Propositions 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9 can be verified by a routine proof.

Proposition 1.4 (The operations of a UP-algebra $\mathcal{P}(X)$ is left distributive over the operations of a semigroup $\mathcal{P}(X)$)**.** *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $A \cdot (B \cap C) = (A \cdot B) \cap (A \cdot C)$,

(2) $A \cdot (B \cup C) = (A \cdot B) \cup (A \cdot C)$,

(3) $A * (B \cap C) = (A * B) \cap (A * C)$,

(4) $A * (B \cup C) = (A * B) \cup (A * C)$,

(5) $A \cdot (B \otimes C) = (A \cdot B) \otimes (A \cdot C)$,

(6) $A * (B \odot C) = (A * B) \odot (A * C)$,

(7) $A \cdot (B \boxdot C) = (A \cdot B) \boxdot (A \cdot C)$,

(8) $A * (B \boxdot C) = (A * B) \boxdot (A * C)$,

(9) $A \cdot (B \boxtimes C) = (A \cdot B) \boxtimes (A \cdot C)$, *and*

(10) $A * (B \boxtimes C) = (A * B) \boxtimes (A * C)$.

Proposition 1.5 (The operations of a UP-algebra $\mathcal{P}(X)$ is right distributive over the operations of a semigroup $\mathcal{P}(X)$)**.** *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $(A \boxdot B) \cdot C = (A \cdot C) \boxdot (B \cdot C)$,

(2) $(A \boxdot B) * C = (A * C) \boxdot (B * C)$,

(3) $(A \boxtimes B) \cdot C = (A \cdot C) \boxtimes (B \cdot C)$, *and*

(4) $(A \boxtimes B) * C = (A * C) \boxtimes (B * C)$.

Proposition 1.6 (The operations of a semigroup $\mathcal{P}(X)$ is left distributive over the operations of a UP-algebra $\mathcal{P}(X)$)**.** *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $A \odot (B * C) = (A \odot B) * (A \odot C)$,

(2) $A \otimes (B \cdot C) = (A \otimes B) \cdot (A \otimes C)$,

(3) $A \boxdot (B \cdot C) = (A \boxdot B) \cdot (A \boxdot C)$, *and*

(4) $A \boxdot (B * C) = (A \boxdot B) * (A \boxdot C)$.

Proposition 1.7 (The operations of a semigroup $\mathcal{P}(X)$ is right distributive over the operations of a UP-algebra $\mathcal{P}(X)$)**.** *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $(A * B) \odot C = (A \odot C) * (B \odot C)$,

(2) $(A \cdot B) \otimes C = (A \otimes C) \cdot (B \otimes C)$,

(3) $(A \cdot B) \boxtimes C = (A \boxtimes C) \cdot (B \boxtimes C)$, *and*

(4) $(A * B) \boxtimes C = (A \boxtimes C) * (B \boxtimes C)$.

Proposition 1.8. *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $(A \cap B) \cdot C = (A \cdot C) \cup (B \cdot C)$,

(2) $(A \cup B) \cdot C = (A \cdot C) \cap (B \cdot C)$,

(3) $(A \cap B) * C = (A * C) \cup (B * C)$,

(4) $(A \cup B) * C = (A * C) \cap (B * C)$,

(5) $(A \odot B) \cdot C = (A \cdot C) \otimes (B \cdot C)$, *and*

(6) $(A \otimes B) * C = (A * C) \odot (B * C)$.

Proposition 1.9. *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

(1) $(A \cdot B) \odot C = (A \otimes C) * (B \otimes C)$, *and*

(2) $(A * B) \otimes C = (A \odot C) \cdot (B \odot C)$.

Proposition 1.10. *Let $A = (A, \cdot, *, 0)$ be an algebra of type $(2, 2, 0)$ in which $(A, \cdot, 0)$ is a UP-algebra and $(A, *)$ is a semigroup. Then the following properties hold:*

(1) *if A is an l-UP-semigroup, then $x * 0 = 0$ for all $x \in A$,*

(2) *if A is an r-UP-semigroup, then $0 * x = 0$ for all $x \in A$,*

(3) *if the operation "$\cdot$" is right distributive over the operation "$*$", then $x*x = x$ for all $x \in A$, and*

(4) *$A = \{0\}$ is one and only one (r, f)-UP-semigroup and (f, f)-UP-semigroup.*

Proof. (1) Assume that A is an l-UP-semigroup. Then, by (1.1), we have

$$x * 0 = x * (0 \cdot 0) = (x * 0) \cdot (x * 0) = 0 \text{ for all } x \in A.$$

(2) Assume that A is an r-UP-semigroup. Then, by (1.1), we have

$$0 * x = (0 \cdot 0) * x = (0 * x) \cdot (0 * x) = 0 \text{ for all } x \in A.$$

(3) Assume that the operation "$\cdot$" is right distributive over the operation "$*$". Then, by (UP-3), we have

$$0 = (0 * 0) \cdot 0 = (0 \cdot 0) * (0 \cdot 0) = 0 * 0.$$

Thus, by (UP-2), we have

$$x = 0 \cdot x = (0 * 0) \cdot x = (0 \cdot x) * (0 \cdot x) = x * x \text{ for all } x \in A.$$

(4) By (UP-2), (1.1), (1) and (2), we have

$$x = 0 \cdot x = (x * 0) \cdot x = (x \cdot x) * (0 \cdot x) = 0 * x = 0 \text{ for all } x \in A.$$

Hence, $A = \{0\}$ is one and only one (r, f)-UP-semigroup and (f, f)-UP-semigroup. ■

Example 1.11. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

and

$*$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0

Then $(A, \cdot, *, 0)$ is an f-UP-semigroup.

Let X be a universal set. Then, by above propositions and an example, we get:

Types of algebras	**Examples**
l-UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Proposition 1.6 (1))
	$(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Proposition 1.6 (2))
	$(\mathcal{P}(X), \cdot, \boxdot, \emptyset)$ (see Proposition 1.6 (3))
	$(\mathcal{P}(X), *, \boxdot, X)$ (see Proposition 1.6 (4))
r-UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Proposition 1.7 (1))
	$(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Proposition 1.7 (2))
	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Proposition 1.7 (3))
	$(\mathcal{P}(X), *, \boxtimes, X)$ (see Proposition 1.7 (4))
f-UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.6 (1) and 1.7 (1))
	$(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.6 (2) and 1.7 (2))
	$(A, \cdot, *, 0)$ (see Example 1.11)
(l, l)-UP-semigroup	$(\mathcal{P}(X), \cdot, \boxdot, \emptyset)$ (see Propositions 1.6 (3) and 1.4 (7))
	$(\mathcal{P}(X), *, \boxdot, X)$ (see Propositions 1.6 (4) and 1.4 (8))
(r, l)-UP-semigroup	$(\mathcal{P}(X), \cdot, \boxdot, \emptyset)$ (see Propositions 1.6 (3) and 1.5 (1))
	$(\mathcal{P}(X), *, \boxdot, X)$ (see Propositions 1.6 (4) and 1.5 (2))
(l, r)-UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.7 (1) and 1.4 (6))
	$(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.7 (2) and 1.4 (5))
	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3) and 1.4 (9))
	$(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4) and 1.4 (10))
(r, r)-UP-semigroup	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3) and 1.5 (3))
	$(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4) and 1.5 (4))
(f, l)-UP-semigroup	$(\mathcal{P}(X), \cdot, \boxdot, \emptyset)$ (see Propositions 1.6 (3), 1.4 (7), and 1.5 (1))
	$(\mathcal{P}(X), *, \boxdot, X)$ (see Propositions 1.6 (4), 1.4 (8), and 1.5 (2))
(f, r)-UP-semigroup	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3), 1.4 (9), and 1.5 (3))
	$(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4), 1.4 (10), and 1.5 (4))
(l, f)-UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.6 (1), 1.4 (6), and 1.7 (1))
	$(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.6 (2), 1.4 (5), and 1.7 (2))
(r, f)-UP-semigroup	$\{0\}$ is one and only one (r, f)-UP-semigroup
(f, f)-UP-semigroup	$\{0\}$ is one and only one (f, f)-UP-semigroup

Hence, we have the following diagram:

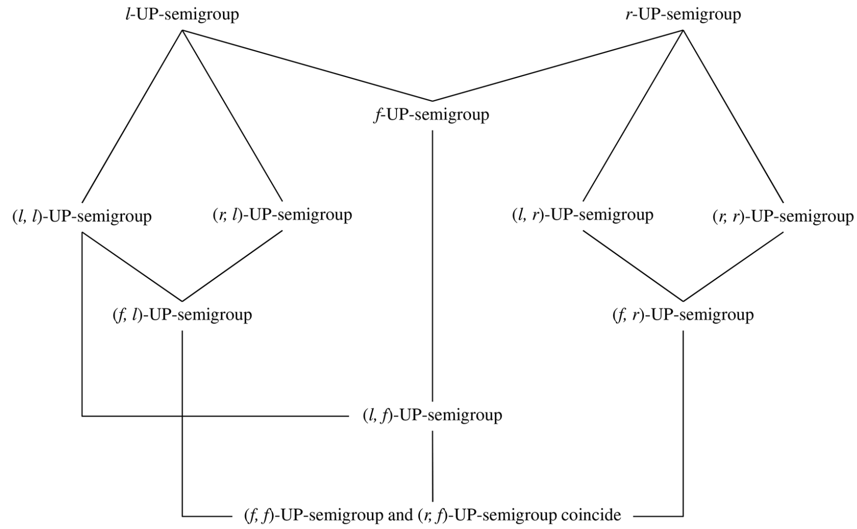

Figure 1. New algebras of type (2,2,0).

Conclusion

We have introduced the notions of left UP-semigroups, right UP-semigroups, fully UP-semigroups, left-left UP-semigroups, right-left UP-semigroups, left-right UP-semigroups, right-right UP-semigroups, fully-left UP-semigroups, fully-right UP-semigroups, left-fully UP-semigroups, right-fully UP-semigroups and fully-fully UP-semigroups, and have found examples. We have that right-fully UP-semigroups and fully-fully UP-semigroups coincide, and it is only $\{0\}$. In further study, we will apply the notion of fuzzy sets and fuzzy soft sets to the theory of all above notions.

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