

INTRODUCING FULLY UP-SEMIGROUPS¹

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Abstract

In this paper, we introduce some new classes of algebras related to UP-algebras and semigroups, called a left UP-semigroup, a right UP-semigroup, a fully UP-semigroup, a left-left UP-semigroup, a right-left UP-semigroup, a left-right UP-semigroup, a right-right UP-semigroup, a fully-left UP-semigroup, a fully-right UP-semigroup, a left-fully UP-semigroup, a right-fully UP-semigroup, a fully-fully UP-semigroup, and find their examples.

Keywords: semigroup, UP-algebra, fully UP-semigroup.

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1. INTRODUCTION AND PRELIMINARIES

In the literature, several researchers introduced a new class of algebras related to logical algebras and semigroups such as: In 1993, Jun, Hong and Roh [4] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup. In 1998, Jun, Xin, and Roh [5, 6] renamed the BCI-semigroup as the IS-algebra and studied further properties of these algebras. In 2006, Kim [8] introduced the notion of KS-semigroups. In 2011, Ahn and Kim [1] introduced the notion of BE-semigroups. In 2015, Endam and Vilela [2] introduced the notion of JB-semigroups. In 2016, Sultana and Chaudhary [11] introduced the notion of BCH-semigroups. In 2018, Kareem and Hasan introduced and analyzed the concept of KU-semigroups in the recently published article [7]. It is known that UP-algebra is a generalization of KU-algebra [3]. Several authors also studied the algebraic structures with semigroups (see, for example: [1, 8–11]).

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In this paper, we introduce some new classes of algebras related to UP-algebras and semigroups, called a left UP-semigroup, a right UP-semigroup, a fully UP-semigroup, a left-left UP-semigroup, a right-left UP-semigroup, a left-right UP-semigroup, a right-right UP-semigroup, a fully-left UP-semigroup, a fully-right UP-semigroup, a left-fully UP-semigroup, a right-fully UP-semigroup, a fully-fully UP-semigroup, and find their examples.

Before we begin our study, we will introduce the definition of a UP-algebra.

Definition 1.1 [3]. An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where A is a nonempty set, \cdot is a binary operation on A , and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms: for any $x, y, z \in A$,

$$\text{(UP-1)} \quad (y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0,$$

$$\text{(UP-2)} \quad 0 \cdot x = x,$$

$$\text{(UP-3)} \quad x \cdot 0 = 0, \text{ and}$$

$$\text{(UP-4)} \quad x \cdot y = 0 \text{ and } y \cdot x = 0 \text{ imply } x = y.$$

Proposition 1.2. *In a UP-algebra $A = (A, \cdot, 0)$, the following assertions are valid ((1.1)–(1.7), see [3], Proposition 1.7).*

$$(1.1) \quad (\forall x \in A)(x \cdot x = 0),$$

$$(1.2) \quad (\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0),$$

$$(1.3) \quad (\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0),$$

$$(1.4) \quad (\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0),$$

$$(1.5) \quad (\forall x, y \in A)(x \cdot (y \cdot x) = 0),$$

$$(1.6) \quad (\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x),$$

$$(1.7) \quad (\forall x, y \in A)(x \cdot (y \cdot y) = 0),$$

$$(1.8) \quad (\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z)))) = 0),$$

$$(1.9) \quad (\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0),$$

$$(1.10) \quad (\forall x, y, z \in A)((x \cdot y) \cdot z) \cdot (y \cdot z) = 0),$$

$$(1.11) \quad (\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0),$$

$$(1.12) \quad (\forall x, y, z \in A)((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and}$$

$$(1.13) \quad (\forall a, x, y, z \in A)((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0).$$

Proof. (1.8) By (UP-1), we have $(y \cdot z) \cdot ((a \cdot y) \cdot (a \cdot z)) = 0$. By (1.3), we have

$$(x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0.$$

(1.9) By (UP-1), we have $(x \cdot y) \cdot ((a \cdot x) \cdot (a \cdot y)) = 0$. By (1.4), we have

$$(((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0.$$

(1.10) Now,

$$\begin{aligned}
 ((1.9)) \quad & 0 = (((x \cdot 0) \cdot (x \cdot y)) \cdot z) \cdot ((0 \cdot y) \cdot z) \\
 ((UP-2), (UP-3)) \quad & = ((0 \cdot (x \cdot y)) \cdot z) \cdot (y \cdot z) \\
 ((UP-2)) \quad & = ((x \cdot y) \cdot z) \cdot (y \cdot z).
 \end{aligned}$$

Hence, $((x \cdot y) \cdot z) \cdot (y \cdot z) = 0$.

(1.11) Assume that $x \cdot y = 0$. By (1.3), we have $(z \cdot x) \cdot (z \cdot y) = 0$. By (1.10) and (UP-2), we have

$$x \cdot (z \cdot y) = 0 \cdot (x \cdot (z \cdot y)) = ((z \cdot x) \cdot (z \cdot y)) \cdot (x \cdot (z \cdot y)) = 0.$$

Hence, $x \cdot (z \cdot y) = 0$.

(1.12) By (1.10), we have

$$((x \cdot y) \cdot z) \cdot (y \cdot z) = 0.$$

By (1.5), we have

$$(y \cdot z) \cdot (x \cdot (y \cdot z)) = 0.$$

It follows from (1.2) that $((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0$.

(1.13) By (1.5), we have $y \cdot (x \cdot y) = 0$ and $(x \cdot y) \cdot (a \cdot (x \cdot y)) = 0$. By (1.2), we have $y \cdot (a \cdot (x \cdot y)) = 0$. By (1.4), we have

$$((a \cdot (x \cdot y)) \cdot (a \cdot z)) \cdot (y \cdot (a \cdot z)) = 0.$$

By (UP-1), we have

$$((x \cdot y) \cdot z) \cdot ((a \cdot (x \cdot y)) \cdot (a \cdot z)) = 0.$$

It follows from (1.2) that $((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0$. ■

Let X be a universal set. Define two binary operations \cdot and $*$ on the power set of X by putting, for all $A, B \in \mathcal{P}(X)$,

$$(1.14) \quad A \cdot B = A' \cap B,$$

$$(1.15) \quad A * B = A' \cup B$$

where A' means the complement of a subset A . Then $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1* [3], Example 1.4, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2* [3], Example 1.5.

Now, define four binary operations $\odot, \otimes, \sqsupseteq$ and \boxtimes on the power set of X by putting, for all $A, B \in \mathcal{P}(X)$,

$$(1.16) \quad A \odot B = X,$$

$$(1.17) \quad A \otimes B = \emptyset,$$

$$(1.18) \quad A \sqsupseteq B = B,$$

$$(1.19) \quad A \boxtimes B = A.$$

Then $(\mathcal{P}(X), \odot), (\mathcal{P}(X), \otimes), (\mathcal{P}(X), \sqsupseteq)$ and $(\mathcal{P}(X), \boxtimes)$ are semigroups which is determined by direct verification. Furthermore, we know that $(\mathcal{P}(X), \cap, X)$ and $(\mathcal{P}(X), \cup, \emptyset)$ are monoids.

Definition 1.3. Let A be a nonempty set, \cdot and $*$ are binary operations on A , and 0 is a fixed element of A (i.e., a nullary operation). An algebra $A = (A, \cdot, *, 0)$ of type $(2, 2, 0)$ in which $(A, \cdot, 0)$ is a UP-algebra and $(A, *)$ is a semigroup is called

- (1) a *left UP-semigroup* (in short, an *l-UP-semigroup*) if the operation “ $*$ ” is left distributive over the operation “ \cdot ”,
- (2) a *right UP-semigroup* (in short, an *r-UP-semigroup*) if the operation “ $*$ ” is right distributive over the operation “ \cdot ”,
- (3) a *fully UP-semigroup* (in short, an *f-UP-semigroup*) if the operation “ $*$ ” is distributive (on both sides) over the operation “ \cdot ”,
- (4) a *left-left UP-semigroup* (in short, an *(l, l)-UP-semigroup*) if the operation “ \cdot ” is left distributive over the operation “ $*$ ” and the operation “ $*$ ” is left distributive over the operation “ \cdot ”,
- (5) a *right-left UP-semigroup* (in short, an *(r, l)-UP-semigroup*) if the operation “ \cdot ” is right distributive over the operation “ $*$ ” and the operation “ $*$ ” is left distributive over the operation “ \cdot ”,
- (6) a *left-right UP-semigroup* (in short, an *(l, r)-UP-semigroup*) if the operation “ \cdot ” is left distributive over the operation “ $*$ ” and the operation “ $*$ ” is right distributive over the operation “ \cdot ”,
- (7) a *right-right UP-semigroup* (in short, an *(r, r)-UP-semigroup*) if the operation “ \cdot ” is right distributive over the operation “ $*$ ” and the operation “ $*$ ” is right distributive over the operation “ \cdot ”,
- (8) a *fully-left UP-semigroup* (in short, an *(f, l)-UP-semigroup*) if the operation “ \cdot ” is distributive (on both sides) over the operation “ $*$ ” and the operation “ $*$ ” is left distributive over the operation “ \cdot ”,
- (9) a *fully-right UP-semigroup* (in short, an *(f, r)-UP-semigroup*) if the operation “ \cdot ” is distributive (on both sides) over the operation “ $*$ ” and the operation “ $*$ ” is right distributive over the operation “ \cdot ”,

- (10) a *left-fully UP-semigroup* (in short, an (l, f) -UP-semigroup) if the operation “ \cdot ” is left distributive over the operation “ $*$ ” and the operation “ $*$ ” is distributive (on both sides) over the operation “ \cdot ”,
- (11) a *right-fully UP-semigroup* (in short, an (r, f) -UP-semigroup) if the operation “ \cdot ” is right distributive over the operation “ $*$ ” and the operation “ $*$ ” is distributive (on both sides) over the operation “ \cdot ”, and
- (12) a *fully-fully UP-semigroup* (in short, an (f, f) -UP-semigroup) if the operation “ \cdot ” is distributive (on both sides) over the operation “ $*$ ” and the operation “ $*$ ” is distributive (on both sides) over the operation “ \cdot ”.

In what follows, let A and B denote UP-algebras unless otherwise specified. The following proposition is very important for the study of UP-algebras.

The proof of Propositions 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9 can be verified by a routine proof.

Proposition 1.4 (The operations of a UP-algebra $\mathcal{P}(X)$ is left distributive over the operations of a semigroup $\mathcal{P}(X)$). *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

- (1) $A \cdot (B \cap C) = (A \cdot B) \cap (A \cdot C)$,
- (2) $A \cdot (B \cup C) = (A \cdot B) \cup (A \cdot C)$,
- (3) $A * (B \cap C) = (A * B) \cap (A * C)$,
- (4) $A * (B \cup C) = (A * B) \cup (A * C)$,
- (5) $A \cdot (B \otimes C) = (A \cdot B) \otimes (A \cdot C)$,
- (6) $A * (B \odot C) = (A * B) \odot (A * C)$,
- (7) $A \cdot (B \boxplus C) = (A \cdot B) \boxplus (A \cdot C)$,
- (8) $A * (B \boxplus C) = (A * B) \boxplus (A * C)$,
- (9) $A \cdot (B \boxtimes C) = (A \cdot B) \boxtimes (A \cdot C)$, and
- (10) $A * (B \boxtimes C) = (A * B) \boxtimes (A * C)$.

Proposition 1.5 (The operations of a UP-algebra $\mathcal{P}(X)$ is right distributive over the operations of a semigroup $\mathcal{P}(X)$). *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

- (1) $(A \boxplus B) \cdot C = (A \cdot C) \boxplus (B \cdot C)$,
- (2) $(A \boxplus B) * C = (A * C) \boxplus (B * C)$,

$$(3) (A \boxtimes B) \cdot C = (A \cdot C) \boxtimes (B \cdot C), \text{ and}$$

$$(4) (A \boxtimes B) * C = (A * C) \boxtimes (B * C).$$

Proposition 1.6 (The operations of a semigroup $\mathcal{P}(X)$ is left distributive over the operations of a UP-algebra $\mathcal{P}(X)$). *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

$$(1) A \odot (B * C) = (A \odot B) * (A \odot C),$$

$$(2) A \otimes (B \cdot C) = (A \otimes B) \cdot (A \otimes C),$$

$$(3) A \square (B \cdot C) = (A \square B) \cdot (A \square C), \text{ and}$$

$$(4) A \square (B * C) = (A \square B) * (A \square C).$$

Proposition 1.7 (The operations of a semigroup $\mathcal{P}(X)$ is right distributive over the operations of a UP-algebra $\mathcal{P}(X)$). *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

$$(1) (A * B) \odot C = (A \odot C) * (B \odot C),$$

$$(2) (A \cdot B) \otimes C = (A \otimes C) \cdot (B \otimes C),$$

$$(3) (A \cdot B) \boxtimes C = (A \boxtimes C) \cdot (B \boxtimes C), \text{ and}$$

$$(4) (A * B) \boxtimes C = (A \boxtimes C) * (B \boxtimes C).$$

Proposition 1.8. *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

$$(1) (A \cap B) \cdot C = (A \cdot C) \cup (B \cdot C),$$

$$(2) (A \cup B) \cdot C = (A \cdot C) \cap (B \cdot C),$$

$$(3) (A \cap B) * C = (A * C) \cup (B * C),$$

$$(4) (A \cup B) * C = (A * C) \cap (B * C),$$

$$(5) (A \odot B) \cdot C = (A \cdot C) \otimes (B \cdot C), \text{ and}$$

$$(6) (A \otimes B) * C = (A * C) \odot (B * C).$$

Proposition 1.9. *Let X be a universal set. Then the following properties hold: for any $A, B, C \in \mathcal{P}(X)$,*

$$(1) (A \cdot B) \odot C = (A \otimes C) * (B \otimes C), \text{ and}$$

$$(2) (A * B) \otimes C = (A \odot C) \cdot (B \odot C).$$

Proposition 1.10. *Let $A = (A, \cdot, *, 0)$ be an algebra of type $(2, 2, 0)$ in which $(A, \cdot, 0)$ is a UP-algebra and $(A, *)$ is a semigroup. Then the following properties hold:*

- (1) *if A is an l -UP-semigroup, then $x * 0 = 0$ for all $x \in A$,*
- (2) *if A is an r -UP-semigroup, then $0 * x = 0$ for all $x \in A$,*
- (3) *if the operation “ \cdot ” is right distributive over the operation “ $*$ ”, then $x * x = x$ for all $x \in A$, and*
- (4) *$A = \{0\}$ is one and only one (r, f) -UP-semigroup and (f, f) -UP-semigroup.*

Proof. (1) Assume that A is an l -UP-semigroup. Then, by (1.1), we have

$$x * 0 = x * (0 \cdot 0) = (x * 0) \cdot (x * 0) = 0 \text{ for all } x \in A.$$

(2) Assume that A is an r -UP-semigroup. Then, by (1.1), we have

$$0 * x = (0 \cdot 0) * x = (0 * x) \cdot (0 * x) = 0 \text{ for all } x \in A.$$

(3) Assume that the operation “ \cdot ” is right distributive over the operation “ $*$ ”. Then, by (UP-3), we have

$$0 = (0 * 0) \cdot 0 = (0 \cdot 0) * (0 \cdot 0) = 0 * 0.$$

Thus, by (UP-2), we have

$$x = 0 \cdot x = (0 * 0) \cdot x = (0 \cdot x) * (0 \cdot x) = x * x \text{ for all } x \in A.$$

(4) By (UP-2), (1.1), (1) and (2), we have

$$x = 0 \cdot x = (x * 0) \cdot x = (x \cdot x) * (0 \cdot x) = 0 * x = 0 \text{ for all } x \in A.$$

Hence, $A = \{0\}$ is one and only one (r, f) -UP-semigroup and (f, f) -UP-semigroup. ■

Example 1.11. Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3	and	$*$	0	1	2	3
0	0	1	2	3		0	0	0	0	0
1	0	0	2	3		1	0	0	0	0
2	0	1	0	3		2	0	0	0	1
3	0	1	2	0		3	0	0	1	0

Then $(A, \cdot, *, 0)$ is an f -UP-semigroup.

Let X be a universal set. Then, by above propositions and an example, we get:

Types of algebras	Examples
l -UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Proposition 1.6 (1)) $(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Proposition 1.6 (2)) $(\mathcal{P}(X), \cdot, \square, \emptyset)$ (see Proposition 1.6 (3))
r -UP-semigroup	$(\mathcal{P}(X), *, \square, X)$ (see Proposition 1.6 (4)) $(\mathcal{P}(X), *, \odot, X)$ (see Proposition 1.7 (1)) $(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Proposition 1.7 (2)) $(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Proposition 1.7 (3)) $(\mathcal{P}(X), *, \boxtimes, X)$ (see Proposition 1.7 (4))
f -UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.6 (1) and 1.7 (1)) $(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.6 (2) and 1.7 (2)) $(A, \cdot, *, 0)$ (see Example 1.11)
(l, l) -UP-semigroup	$(\mathcal{P}(X), \cdot, \square, \emptyset)$ (see Propositions 1.6 (3) and 1.4 (7)) $(\mathcal{P}(X), *, \square, X)$ (see Propositions 1.6 (4) and 1.4 (8))
(r, l) -UP-semigroup	$(\mathcal{P}(X), \cdot, \square, \emptyset)$ (see Propositions 1.6 (3) and 1.5 (1)) $(\mathcal{P}(X), *, \square, X)$ (see Propositions 1.6 (4) and 1.5 (2))
(l, r) -UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.7 (1) and 1.4 (6)) $(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.7 (2) and 1.4 (5)) $(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3) and 1.4 (9)) $(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4) and 1.4 (10))
(r, r) -UP-semigroup	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3) and 1.5 (3)) $(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4) and 1.5 (4))
(f, l) -UP-semigroup	$(\mathcal{P}(X), \cdot, \square, \emptyset)$ (see Propositions 1.6 (3), 1.4 (7), and 1.5 (1)) $(\mathcal{P}(X), *, \square, X)$ (see Propositions 1.6 (4), 1.4 (8), and 1.5 (2))
(f, r) -UP-semigroup	$(\mathcal{P}(X), \cdot, \boxtimes, \emptyset)$ (see Propositions 1.7 (3), 1.4 (9), and 1.5 (3)) $(\mathcal{P}(X), *, \boxtimes, X)$ (see Propositions 1.7 (4), 1.4 (10), and 1.5 (4))
(l, f) -UP-semigroup	$(\mathcal{P}(X), *, \odot, X)$ (see Propositions 1.6 (1), 1.4 (6), and 1.7 (1)) $(\mathcal{P}(X), \cdot, \otimes, \emptyset)$ (see Propositions 1.6 (2), 1.4 (5), and 1.7 (2))
(r, f) -UP-semigroup	$\{0\}$ is one and only one (r, f) -UP-semigroup
(f, f) -UP-semigroup	$\{0\}$ is one and only one (f, f) -UP-semigroup

Hence, we have the following diagram:

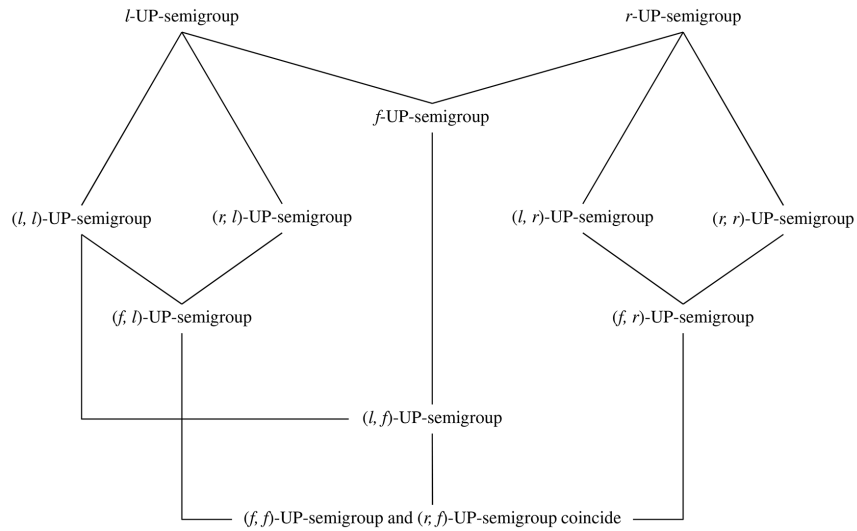


Figure 1. New algebras of type (2,2,0).

CONCLUSION

We have introduced the notions of left UP-semigroups, right UP-semigroups, fully UP-semigroups, left-left UP-semigroups, right-left UP-semigroups, left-right UP-semigroups, right-right UP-semigroups, fully-left UP-semigroups, fully-right UP-semigroups, left-fully UP-semigroups, right-fully UP-semigroups and fully-fully UP-semigroups, and have found examples. We have that right-fully UP-semigroups and fully-fully UP-semigroups coincide, and it is only $\{0\}$. In further study, we will apply the notion of fuzzy sets and fuzzy soft sets to the theory of all above notions.

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