

SOME PROPERTIES OF THE ZERO DIVISOR GRAPH OF A COMMUTATIVE RING

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Abstract

Let $\Gamma(R)$ be the zero divisor graph for a commutative ring with identity. The k -domination number and the 2-packing number of $\Gamma(R)$, where R is an Artinian ring, are computed. k -dominating sets and 2-packing sets for the zero divisor graph of the ring of Gaussian integers modulo n , $\Gamma(\mathbb{Z}_n[i])$, are constructed. The center, the median, the core, as well as the automorphism group of $\Gamma(\mathbb{Z}_n[i])$ are determined. Perfect zero divisor graphs $\Gamma(R)$ are investigated.

Keywords: automorphism group of a graph, center of a graph, core of a graph, k -domination number, Gaussian integers modulo n , median of a graph, 2-packing, perfect graph, and zero divisor graph.

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