Discussiones Mathematicae
General Algebra and Applications 33 (2013) 221–231
doi:10.7151/dmgaa.1202

VAGUE IDEALS OF IMPLICATION GROUPOIDS

Ravi Kumar Bandaru[1]

Department of Engineering Mathematics
GITAM University, Hyderabad Campus
Andhra Pradesh, India-502329
e-mail: ravimaths83@gmail.com

and

K.P. Shum

Insititute of Mathematics
Yunnan University, Kunming-650091, China
e-mail: kpshum@ynu.edu.cn

Abstract

We introduce the concept of vague ideals in a distributive implication groupoid and investigate their properties. The vague ideals of a distributive implication groupoid are also characterized.

Keywords: implication groupoids, distributive implication groupiods, vague ideals.

2010 Mathematics Subject Classification: 06F35, 03G25.

1. Introduction

I. Chajda and R. Halas [5] first introduced the concept of an implication groupoid as a generalization of the implication reduct of intuitionistic logic, i.e., a Hilbert algebra [1, 4] and studied some connections among ideals, deductive systems and congruence kernels whenever implication groupoid is distributive. In [6, 8], Y.B. Jun *et. al* introduced the concept of fuzzy ideal, fuzzy deductive systems in Hilbert algebras and discuss the relation between fuzzy ideals and fuzzy deductive systems. It is noticed that fuzzy algebra is now a well established branch of study

[1]Corresponding author.

[10]. Recently W.L. Gau, and D.J. Buehrer [7] proposed the theory of vague sets as an improvement of theory of fuzzy sets in approximating the real life situations. According to Gau and Buehrer,a vague set A in the Universe of discourse U is a pair (t_A, f_A) where t_A and f_A are fuzzy subsets of U satisfying the condition $t_A \leq 1 - f_A$, i.e., $t_A(u) \leq 1 - f_A(u)$ for all $u \in U$. Ranjit Biswas [3] initiated the study of vague algebra by studying vague groups, vague normal groups and the properties related to them. In [9], Jun and Park introduced the concept of vague ideals in a subtraction algebra and studied their properties.

In this paper we use the notion of vague set to study the vague ideals of a distributive implication groupoid and then we obtain some related results.

2. Preliminaries

In this section, we present some preliminaries on the theory of vague sets (VS). In his pioneer work [11], Zadeh proposed the theory of fuzzy sets. Since then, the fuzzy sets have been applied in wide varieties of fields like Computer Science, Management Science, Medical Sciences, Engineering problems, etc. to list a few only. Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universe of discourse. The membership function for fuzzy sets can take any value from the closed interval $[0, 1]$. An fuzzy set A is defined as the set of ordered pairs $A = \{(u, \mu_A(u)) \mid u \in U\}$ where $\mu_A(u)$ is the grade of membership of element u in set A. The greater $\mu_A(u)$ is the greater of the truth of the statement that the element u belongs to the set A. But Gau and Buehrer [7] pointed out that this single value combines the vidence for u and the evidence against u. It does not indicate the vidence for u and the vidence against u, and it does not also indicate how much there is of each. Consequently, there is a genuine necessity of a different kind of fuzzy sets which could be treated as a generalization of the fuzzy sets proposed by Zadeh in [11].

Definition 2.1. A vague set A in the universe of discourse U is characterized by two membership functions given by:

(1) A truth membership function $t_A : U \to [0, 1]$ and
(2) A false membership function $f_A : U \to [0, 1]$

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the "evidence for u", and $f_A(u)$ is a lower bound on the negation of u derived from the "evidence against u", and $t_A(u) + f_A(u) \leq 1$.

Thus the grade of membership of u in the vague set A is bounded by a sub interval $[t_A(u), 1 - f_A(u)]$ of $[0, 1]$. This indicates that if the actual grade of membership is $\mu(u)$, then

$$t_A(u) \leq \mu(u) \leq 1 - f_A(u).$$

The vague set A is written as

$$A = \{(u, [t_A(u), f_A(u)]) \mid u \in U\},$$

where the interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A and is denoted by $V_A(u)$.

It is worth to mention here that the interval-valued fuzzy sets (i-v fuzzy sets) are not vague sets. In the (i-v fuzzy sets), an interval valued membership value is assigned to each element of the universe considering the vidence for u without considering the vidence against u. In vague sets both values are independent proposed by the decision maker. This makes a major difference in the judgment about the grade of membership.

We now give the following crucial definition.

Definition 2.2. A vague set A of a set U is called

(1) the zero vague set of U if $t_A(u) = 0$ and $f_A(u) = 1$ for all $u \in U$,

(2) the unit vague set of U if $t_A(u) = 1$ and $f_A(u) = 0$ for all $u \in U$.

(3) the α-vague set of U if $t_A(u) = \alpha$ and $f_A(u) = 1 - \alpha$ for all $u \in U$, where $\alpha \in (0, 1)$.

For $\alpha, \beta \in [0, 1]$ we now define the (α, β)-cut and the α-cut of a vague set.

Definition 2.3. Let A be a vague set of a universe X with the true-membership function t_A and the false-membership function f_A. The (α, β)-cut of the vague set A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by

$$A_{(\alpha,\beta)} = \{x \in X \mid V_A(x) \geq [\alpha, \beta]\},$$

where $\alpha \leq \beta$. Clearly, $A_{(0,0)} = X$.

The (α, β)-cuts are also called the vague-cuts of the vague set A.

We give the following definitions.

Definition 2.4. The α-cut of the vague set A is a crisp subset A_α of the set X given by $A_\alpha = A_{(\alpha,\alpha)}$.

Note that $A_0 = X$, and if $\alpha \geq \beta$, then $A_\beta \subseteq A_\alpha$ and $A_{(\alpha,\beta)} = A_\alpha$. Equivalently, we can define the α-cut as

$$A_\alpha = \{x \in X \mid t_A(x) \geq \alpha\}.$$

For our discussion, we shall use the following notations which were given in [3] on interval arithmetic.

$I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. Similarly, we understand the relations $I_1 \leq I_2$ and $I_1 = I_2$. Clearly the relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely. We define the term "imax" to mean the maximum of two intervals as

$$\text{imax}(I_1, I_2) = [\max(a_1, a_2), \max(b_1, b_2)].$$

Similarly, we define "imin". The concept of "imax" and "imin" could be extended to define "isup" and "iinf" of infinite number of elements of $I[0,1]$. It is obvious that $L = \{I[0,1], \text{isup}, \text{iinf}, \leq\}$ is a lattice with universal bounds $[0,0]$ and $[1,1]$.

Let us recall some definitions and results which were discussed in [5, 2] for the development of the paper.

Definition 2.5. An algebra $(X, *, 1)$ of type $(2,0)$ is called an implication groupoid if it satisfies the following identities:

(1) $x * x = 1$
(2) $1 * x = x$ for all $x, y \in X$.

Example 2.6. Let $X = \{1, a, b\}$ in which $*$ is defined by

$*$	1	a	b
1	1	a	b
a	a	1	b
b	a	b	1

Then $(X, *, 1)$ is an implication groupoid.

Definition 2.7. An Implication groupoid $(X, *, 1)$ of type (2,0) is called a distributive implication groupoid if it satisfies the following identity:

$$(LD) \quad x * (y * z) = (x * y) * (x * z) \qquad \textbf{(left distributivity)}$$

for all $x, y, z \in X$.

Example 2.8. Let $X = \{1, a, b, c, d\}$ in which an operation $*$ is defined by

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	b	1
b	1	a	1	1	d
c	1	a	1	1	d
d	1	1	c	c	1

Then $(X, *, 1)$ forms a distributive implication groupoid.

In every implication groupoid, one can introduce the so called induced relation $\leq$ by the setting

$$x \leq y \text{ if and only } x * y = 1.$$

Lemma 2.9. Let $(X, *, 1)$ be a distributive implication groupoid. Then X satisfies the identities

$$x * 1 = 1 \text{ and } x * (y * x) = 1.$$

Moreover, the induced relation $\leq$ is a quasiorder on X and the following relationships are satisfied

(i)	$x \leq 1$	(v)	$y * z \leq (x * y) * (x * z)$
(ii)	$x \leq y * x$	(vi)	$x \leq y$ implies $y * z \leq x * z$
(iii)	$x * ((x * y) * y) = 1$	(vii)	$x * (y * z) \leq y * (x * z)$
(iv)	$1 \leq x$ implies $x = 1$	(viii)	$x * y \leq (y * z) * (x * z)$.

Definition 2.10. Let $\mathcal{X} = (X, *, 1)$ be an implication groupoid. Then, a subset $I \subseteq X$ is called an ideal of $\mathcal{X}$ if

$(I1)$ $1 \in I$,

$(I2)$ $x \in X, y \in I$ imply $x * y \in I$,

$(I3)$ $x \in X, y_1, y_2 \in I$ imply $(y_2 * (y_1 * x)) * x \in I$.

Remark 2.11. If I is an ideal of an implication groupoid $\mathcal{X} = (X, *, 1)$ and $a \in I$, $x \in X$ then $(a * x) * x \in I$.

Definition 2.12. Let $\mathcal{X} = (X, *, 1)$ be an implication groupoid. Then,a subset $D \subseteq X$ is called a deductive system of $\mathcal{X}$ if

$(D1)$ $1 \in D$,

$(D2)$ $x \in D$ and $x * y \in D$ imply $y \in D$.

Theorem 2.13. A nonempty subset I of a distributive implication groupoid $\mathcal{X}$ is an ideal if and only if it is a deductive system of $\mathcal{X}$.

Definition 2.14. Let X be a set. A fuzzy set in X is a function $\mu : X \longrightarrow [0, 1]$.

Definition 2.15. Let μ be a fuzzy set in a set X. For $\alpha \in [0, 1]$, the set $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ is called a level subset of μ.

Definition 2.16. If μ is a fuzzy relation on a set X and ν is a fuzzy set in X, then μ is called a fuzzy relation on ν if

$$\mu(x, y) \leq \min\{\nu(x), \nu(y)\} \text{ for all } x, y \in X.$$

3. Vague ideals

In this section we introduce the concept of a vague ideal in a distributive implication groupoid X.

Throughout this section X is a distributive implication groupoid unless otherwise specified.

We first begin with the following definition.

Definition 3.1. A vague set A of X is called a vague ideal of X if the following conditions hold:

(VI_1) $V_A(1) \geq V_A(x)$, for all $x \in X$.

(VI_2) $V_A(y) \geq \text{imin}\{V_A(x * y), V_A(x)\}$, that is,

$$(VI_1) \qquad t_A(1) \geq t_A(x), 1 - f_A(1) \geq 1 - f_A(x)$$

and

$$(VI_2) \quad t_A(y) \geq \min\{t_A(x * y), t_A(x)\}, 1 - f_A(y) \geq \min\{1 - f_A(x * y), 1 - f_A(x)\}$$

for all $x, y \in X$.

Example 3.2. Let $X = \{1, a, b, c, d\}$ in which $*$ is defined by

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	1	1	d
b	1	1	1	1	d
c	1	1	1	1	d
d	1	a	b	c	1

Then $(X, *, 1)$ is a distributive implication groupoid. Let A be a vague set in X defined as follows:

$$A = \{< 1, [0.7, 0.2] >, < a, [0.5, 0.3] >, < b, [0.5, 0.3] >, \\ < c, [0.5, 0.3] >, < d, [0.7, 0.2] >\}.$$

Then A is a vague ideal of X.

Proposition 3.3. Every vague ideal A of X satisfies

$$x \leq y \Rightarrow V_A(x) \leq V_A(y),$$

for all $x, y \in X$.

Proof. Let $x, y \in X$ and $x \leq y$. Then $x * y = 1$ and so

$$t_A(y) \geq \min\{t_A(x * y), t_A(x)\} = \min\{t_A(1), t_A(x)\} = t_A(x)$$

$$1 - f_A(y) \geq \min\{1 - f_A(x * y), 1 - f_A(x)\} = 1 - f_A(x).$$

Therefore $V_A(x) \leq V_A(y)$. ■

Proposition 3.4. Every vague ideal A of X satisfies

$$(VI_3) \qquad V_A(x * z) \geq \operatorname{imin}\{V_A(x * (y * z)), V_A(y)\}$$

for all $x, y, z \in X$.

Proof. Let $x, y, z \in X$. Then, by Definition 3.1(2) and Proposition 3.3, we deduce that

$$V_A(x * z) \geq \operatorname{imin}\{V_A(y * (x * z)), V_A(y)\} \geq \operatorname{imin}\{V_A(x * (y * z)), V_A(y)\}.$$ ■

The following theorem can be proved easily.

Theorem 3.5. Let A be a vague set in X. Then A is a vague filter of X if and only if it satisfies (VF_1) and (VF_3)

For Vague ideals, we have the following theorem.

Theorem 3.6. Let A be a vague set in X. Then A is a vague ideal of X if and only if it satisfies the following conditions:

(VI_4) $V_A(y * x) \geq V_A(x)$ and
(VI_5) $V_A((a * (b * x)) * x) \geq \operatorname{imin}\{V_A(a), V_A(b)\}$
for all $x, y, a, b \in X$.

Proof. Suppose that A is a vague ideal of X and $x, y, a, b \in X$. Then, by Definition 3.1 and Propositions 3.3, 3.4, we get

$$(VI_4) \quad V_A(y * x) \geq \operatorname{imin}\{V_A(x * (y * x)), V_A(x)\} = \operatorname{imin}\{V_A(1), V_A(x)\} = V_A(x)$$

$$(VI_5) \quad \begin{aligned} V_A((a * (b * x)) * x) &\geq \operatorname{imin}\{V_A((a * (b * x)) * (b * x)), V_A(b)\} \\ &\geq \operatorname{imin}\{V_A(a), V_A(b)\}. \end{aligned}$$

Conversely, assume that the conditions hold. Put $y = x$ in (VI_3), then

$$V_A(1) = V_A(x * x) \geq V_A(x)$$

for all $x \in X$. By (VI_5), we get

$$V_A(y) = V_A(1 * y) = V_A(((x * y) * (x * y)) * y) \geq \text{imin}\{V_A(x * y), V_A(x)\}$$

for all $x, y \in X$. Hence A is a vague ideal of X. ■

Theorem 3.7. Let A be a vague set in X. Then A is a vague ideal of X if and only if it satisfies

$$(VI_6) \qquad z \leq x * y \Rightarrow V_A(y) \geq \text{imin}\{V_A(x), V_A(z)\}$$

for all $x, y, z \in X$.

Proof. Assume that A is a vague ideal of X. Let $x, y, z \in X$ be such that $z \leq x*y$. Then, by Proposition 3.3 and (VI_2), we have

$$V_A(y) \geq \text{imin}\{V_A(x * y), V_A(x)\} \geq \text{imin}\{V_A(z), V_A(x)\}.$$

Conversely, suppose that A satisfies (VI_6). Since $x \leq x * 1$ for all $x \in X$, we have $V_A(1) \geq \text{imin}\{V_A(x), V_A(x)\} = V_A(x)$ by (VI_6). Also, $x \leq (x * y) * y$ for all $x, y \in X$. Hence $V_A(y) \geq \text{imin}\{V_A(x * y), V_A(x)\}$. Therefore A is a vague ideal of X. ■

As a generalization of the above theorem, we have the following theorem.

Theorem 3.8. If a vague set A in X is a vague ideal of X, then

$$(VI_7) \qquad \prod_{i=1}^{n} w_i * x = 1 \Rightarrow V_A(x) \geq \text{imin}\{V_A(w_i) \mid i = 1, 2, \ldots, n\}$$

for all $x, w_1, w_2, \ldots, w_n \in X$, where $\prod_{i=1}^{n} w_i * x = w_n * (w_{n-1} * (\cdots * (w_1 * x) \ldots))$.

Proof. We prove this theorem by using Mathematical induction on n. Let A be a vague ideal of X. Then, by Proposition 3.3 and (VI_6), we know that the condition (VI_7) is valid for $n = 1, 2$. Assume that A satisfies the condition (VI_7) for $n = k$, i.e.,

$$\prod_{i=1}^{k} w_i * x = 1 \Rightarrow \text{imin}\{V_A(w_i) \mid i = 1, \ldots, k\}$$

for all $x, w_1, w_2, \ldots, w_k \in X$. Let $x, w_1, w_2, \ldots, w_k, w_{k+1} \in X$ be such that $\prod_{i=1}^{k+1} w_i * x = 1$. Then

$$V_A(w_i * x) \geq \text{imin}\{V_A(w_j) \mid j = 2, \ldots, k + 1\}.$$

Since A is a vague ideal of X, it follows from (VI_2) that

$$\begin{aligned} V_A(x) &\geq \operatorname{imin}\{V_A(w_1 * x), V_A(w_1)\} \\ &\geq \operatorname{imin}\{V_A(w_1), \{V_A(w_j) \mid j = 2, \ldots, k+1\}\} \\ &= \operatorname{imin}\{V_A(w_j) \mid j = 1, 2, \ldots, k+1\}. \end{aligned}$$

Hence, by mathematical induction, we have proved that A satisfies (VI_7). ■

Theorem 3.9. Let A be a vague set in X satisfying the condition (VI_7). Then A is a vague ideal of X.

Proof. Note that $\underbrace{1 * (1 * (1 * \ldots (1}_{n \text{ times}} * x)) \ldots) = x$. Since $x \leq x * 1$, $V_A(1) \geq V_A(x)$ for all $x \in X$. Thus (VI_1) is valid. Let $x, y, z \in X$ be such that $z \leq x * y$. Then

$$1 = z * (x * y) = z * \underbrace{(1 * \ldots (1 * (1}_{n-2 \text{ times}} * (x * y))) \ldots))$$

and so

$$V_A(y) \geq \operatorname{imin}\{V_A(z), V_A(x), V_A(1)\} = \operatorname{imin}\{V_A(z), V_A(x)\}.$$

Hence by Theorem 3.7, we conclude that A is a vague ideal of X. ■

Theorem 3.10. Let A be a vague ideal of X. Then for any $\alpha, \beta \in [0, 1]$, the vague-cut $A_{(\alpha,\beta)}$ is a crisp ideal of X.

Proof. Obviously, $1 \in A_{(\alpha,\beta)}$. Let $x, y \in X$ be such that $x \in A_{(\alpha,\beta)}$ and $x * y \in A_{(\alpha,\beta)}$. Then $V_A(x) \geq [\alpha, \beta]$, i.e., $t_A(x) \geq \alpha$ and $1 - f_A(x) \geq \beta$ and $V_A(x * y) \geq [\alpha, \beta]$, i.e., $t_A(x * y) \geq \alpha$ and $1 - f_A(x * y) \geq \beta$. It follows from (VI_2) that

$$t_A(y) \geq \min\{t_A(x * y), t_A(x)\} \geq \alpha,$$

$$1 - f_A(y) \geq \min\{1 - f_A(x * y), 1 - f_A(y)\} \geq \beta$$

so that $V_A(y) \geq [\alpha, \beta]$. Hence $y \in A_{(\alpha,\beta)}$ is an ideal of X. ■

The ideal like $A_{(\alpha,\beta)}$ are also called the vague-cut ideals of X. Now, we have the following results.

Proposition 3.11. Let A be a vague ideal of X. Then, two vague-cut ideals $A_{(\alpha,\beta)}$ and $A_{(\omega,\gamma)}$ with $[\alpha, \beta] < [\omega, \gamma]$ are equal if and only if there is no $x \in X$ such that $[\alpha, \beta] \leq V_A(x) \leq [\omega, \gamma]$.

Theorem 3.12. Let X be a finite distributive implication groupoid and A a vague ideal of X. Consider the set $V(A)$ given by

$$V(A) = \{V_A(x) \mid x \in X\}.$$

Then A_i are the only vague-cut ideals of X, where $A_i \in V(A)$.

Proof. Consider $[a_1, a_2] \in I[0,1]$ where $[a_1, a_2] \notin V(A)$. It $[\alpha, \beta] < [a_1, a_2] < [\omega, \gamma]$ where $[\alpha, \beta], [\omega, \gamma] \in V(A)$, then $A_{(\alpha,\beta)} = A_{(a_1,a_2)} = A_{(\omega,\gamma)}$. If $[a_1, a_2] < [a_1, a_3]$ where $[a_1, a_3] = \text{imin}\{V_A(x) \mid x \in X\}$, then $A_{(a_1,a_3)} = X = A_{(a_1,a_2)}$. Hence for any $[a_1, a_2] \in I[0,1]$, the vague-cut ideal $A_{(a_1,a_2)}$ is one of the $A_i \in V(A)$. This completes the proof. ∎

Theorem 3.13. Any ideal I of X is a vague-cut ideal of some vague ideal of X.

Proof. Consider the vague set A of X given by

$$V_A = \begin{cases} [\alpha, \alpha], & \text{if } x \in I \\ [0,0], & \text{if } x \notin I, \end{cases}$$

where $\alpha \in (0,1)$. Since $1 \in I$, we have $V_A(1) = [\alpha, \alpha] \geq V_A(x)$ for all $x \in X$. Let $x, y \in X$. If $y \in I$, then

$$V_A(y) = [\alpha, \alpha] \geq \text{imin}\{V_A(x * y), V_A(x)\}.$$

Assume that $y \notin I$. Then $x \notin I$ or $x * y \notin I$. It follows that

$$V_A(y) = [0, 0] = \text{imin}\{V_A(x * y), V_A(x)\}.$$

Thus A is a vague ideal of X. Clearly $F = A_{(\alpha,\alpha)}$. ∎

Theorem 3.14. Let A be a vague ideal of X. Then the set

$$\mathbb{I} = \{x \in X \mid V_A(x) = V_A(1)\}$$

is a crisp ideal of X.

Proof. Obviously $1 \in \mathbb{I}$. Let $x, y \in X$ be such that $x * y \in \mathbb{I}$ and $x \in \mathbb{I}$. Then $V_A(x * y) = V_A(1) = V_A(x)$ and so

$$V_A(y) \geq \text{imin}\{V_A(x * y), V_A(x)\} = V_A(1)$$

by (VI_1). Since $V_A(1) \geq V_A(y)$ for all $y \in X$, it follows that $V_A(y) = V_A(1)$, and thereby we have $y \in \mathbb{I}$. Therefore $\mathbb{I}$ is a crisp ideal of X. ∎

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Received 4 June 2013
Revised 19 July 2013