

## ORDER OF FINITE SOFT QUASIGROUPS WITH APPLICATION TO EGALITARIANISM

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### Abstract

In this work, a soft set  $(F, A)$  was introduced over a quasigroup  $(Q, \cdot)$  and the study of finite soft quasigroup was carried out, motivated by the study of algebraic structures of soft sets. By introducing the order of a finite soft quasigroup, various inequality relationships that exist between the order of a finite quasigroup, the order of its soft quasigroup and the cardinality of its set of parameters were established. By introducing the arithmetic mean  $\mathcal{AM}(F, A)$  and geometric mean  $\mathcal{GM}(F, A)$  of a finite soft quasigroup  $(F, A)$ , a sort of Lagrange's Formula  $|(F, A)| = |A|\mathcal{AM}(F, A)$  for finite soft quasigroup was gotten. Some of the inequalities gotten gave an upper bound for the order of a finite soft quasigroup in terms of the order of its quasigroup and cardinality of its set of parameters, and a lower bound for the order of the quasigroup in terms of the arithmetic mean of the finite soft quasigroup. A chain of inequalities called the Maclaurin's inequality for

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any finite soft quasigroup  $(F, A)_{(Q, \cdot)}$  was shown to exist. A necessary and sufficient condition for a type of finite soft quasigroup to be extensible to a finite super soft quasigroup was established. This result is of practical use whenever a larger set of parameters is required. The results therein were illustrated with examples. Application to uniformity, equality and equity in distribution for social living is considered.

**Keywords:** soft sets, quasigroups, soft quasigroups, soft subquasigroups, arithmetic and geometric mean, inequalities.

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#### REFERENCES

- [1] A. Albert and A. Baer, *Quasigroups II*, Trans. Amer. Math. Soc. **55** (1944) 401–419.  
<https://doi.org/10.2307/1990259>
- [2] H. Aktas and N. Cagman, *Soft sets and soft groups*, Inf. Sci. **177** (2007) 2726–2735.  
<https://doi.org/10.1016/j.ins.2006.12.008>
- [3] H. Aktas and S. Ozlu, *Cyclic soft groups and their applications on groups*, The Scientific World Journal (2014). Article Id 437324, pp. 5.  
<https://doi.org/10.1155/2014/437324>
- [4] Aristotle, *Nicomachean Ethics*, in: The complete works of Aristotle, ed. J. Barnes, Princeton, Princeton University Press.
- [5] Aristotle, *Politics*, in: The complete works of Aristotle, ed. J. Barnes, Princeton, Princeton University Press.  
<https://doi.org/10.1515/9781400835850-015>
- [6] S. Aslihan and O. Atagun, *Soft groups and normalistic soft groups*, Comput. and Maths with Appl. **62** (2011) 685–698.  
<https://doi.org/10.1016/j.camwa.2011.05.050>
- [7] S. Aslihan, A. Shahzad and M. Adnan, *A new operation on soft sets: Extended difference of soft sets*, J. New Theory **27** (2019) 33–42.
- [8] M. Bronfenbrenner, *Equality and equity*, The Annals of the American Academy of Political and Social Science **409** (1973) 9–23.  
<https://doi.org/10.1177/000271627340900103>
- [9] R.H. Bruck, *Contributions to the theory of loops*, Trans. Amer. Math. Soc. **60** (1946) 245–354.  
<https://doi.org/10.1090/S0002-9947-1946-0017288-3>
- [10] D. Chen, C. Tsang, D. Yeung and X. Wang, *The parameterization reduction of soft sets and its applications*, Comput. Math. Appl. **49** (2005) 757–763.  
<https://doi.org/10.1016/j.camwa.2004.10.036>
- [11] O. Chein, O.H. Pflugfelder and J.D. Smith, *Quasigroups and Loops, Theory and Applications* (Heldermann Verlag, 1990).

- [12] T.G. Jaiyéólá, *Some necessary and sufficient condition for parastrophic invariance of the associative law in quasigroup*, Fasc. Math. **40** (2008) 23–35.
- [13] T.G. Jaiyéólá, *A study of new concepts in smarandache quasigroups and loop*, Pro-Quest Information and Learning (ILQ), Ann Arbor (2009).
- [14] K. Huseyin, O. Akin and A. Emin, *Difference operations of soft matrices with application in decision making*, Punjab Univ. J. Maths. **51** (2019) 1–21.
- [15] K. Maji, A.R. Roy and R. Biswas, *An application of soft sets in a decision making problem*, Comput. Math. Appl. **44** (2002) 1077–1083.  
[https://doi.org/10.1016/S0898-1221\(02\)00215-X](https://doi.org/10.1016/S0898-1221(02)00215-X)
- [16] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl. **37** (1999) 19–31.  
[https://doi.org/10.1016/50898-1221\(99\)00056-5](https://doi.org/10.1016/50898-1221(99)00056-5)
- [17] V.E. Nistala and G. Emandi, *Representation of soft substructures of a soft groups*, IOSR J. Maths. **15** (2019) 41–48.  
<https://doi.org/10.9790/5728-1505034148>
- [18] S. Ayub, M. Shabir and W. Mahmood, *New types of soft rough sets in groups based on normal soft groups*, Comput. Appl. Math. **39** (2) (2020) pp. 15.  
<https://doi.org/10.1007/s40314-020-1098-8>
- [19] B.V. Sai, P.D. Srinivasu and N.V. Murthy, *Soft sets-motivation and overview*, Global J. Pure and Appl. Math. **15** (6) (2019) 1057–1068.  
<https://doi.org/10.37622/GJPAM/15.6.2019.1055-1067>
- [20] A. Sezgin and A.O. Atagun, *On Operations of soft sets*, Comput. Math. Appl. **61** (2011) 1457–1467.  
<https://doi.org/10.1016/j.camwa.2011.01.018>
- [21] Z. Pawlak, *Rough sets*, Int. J. Comp. Inform. Sci. **11** (5) (1982) 341–356.  
<https://doi.org/10.1007/BF01001956>
- [22] H.O. Pflugfelder, *Quasigroups and Loops: Introduction*. Sigma Series in Pure Math. 7 (Heldermann Verlag, Berlin, 1990).
- [23] Z. Ping and W. Qiaoyan, *Operations on soft set*, J. Appl. Math., Article Id 105752 (2013).  
<https://doi.org/10.1155/2013/105752>
- [24] L. Vijayalakshmi and J. Vimala, *On Lattice ordered soft groups*, Int. J. Pure Appl. Math. **12** (2017) 47–55.  
<https://doi.org/10.12732/ijpam.v112i1.3>
- [25] W. Wall, *Drury: Subquasigroups of finite quasigroup*, Pacific J. Math. **7** (4) (1957) 1711–1714.  
<https://projecteuclid.org/euclid.pjm/1103043242>
- [26] L.A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965) 338–353.  
[https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)

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