

STONE COMMUTATOR LATTICES AND BAER RINGS

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Abstract

In this paper, we transfer Davey's characterization for κ -Stone bounded distributive lattices to lattices with certain kinds of quotients, in particular to commutator lattices with certain properties, and obtain related results on prime, radical, complemented and compact elements, annihilators and congruences of these lattices. We then apply these results to certain congruence lattices, in particular to those of semiprime members of semi-degenerate congruence-modular varieties, and use this particular case to transfer Davey's Theorem to commutative unitary rings.

Keywords: (strongly) Stone lattice, commutator lattice, annihilator, modular commutator, Baer ring.

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