Discussiones Mathematicae
General Algebra and Applications 41 (2021) 411–412
https://doi.org/10.7151/dmgaa.1375

REVISITING THE REPRESENTATION THEOREM OF FINITE DISTRIBUTIVE LATTICES WITH PRINCIPAL CONGRUENCES. A *PROOF-BY-PICTURE* APPROACH

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Abstract

A classical result of R.P. Dilworth states that every finite distributive lattice D can be represented as the congruence lattice of a finite lattice L. A sharper form was published in G. Grätzer and E.T. Schmidt in 1962, adding the requirement that all congruences in L be principal. Another variant, published in 1998 by the authors and E.T. Schmidt, constructs a planar semimodular lattice L. In this paper, we merge these two results: we construct L as a planar semimodular lattice in which all congruences are principal. This paper relies on the techniques developed by the authors and E.T. Schmidt in the 1998 paper.

Keywords: principal congruence, finite distributive lattice.

2010 Mathematics Subject Classification: 06B10.

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Received 14 June 2020
Revised 31 October 2020
Accepted 31 October 2020