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FASCINATING NUMBER SEQUENCES FROM FOURTH ORDER DIFFERENCE EQUATION VIA QUATERNION ALGEBRAS

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Abstract

The balancing and Lucas-balancing numbers are solutions of second order recurrence relations. A linear combination of these numbers can also be obtained as solutions of a fourth order recurrence relation. This recurrence relation can be extended to generalized quaternion algebras. Also, the fourth order recurrence relation has application in coding theory.

Keywords: balancing numbers, Lucas-balancing numbers, quaternion algebra, Coding theory, Pell and Lucas-Pell numbers.

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