

GENERALIZED ROUGH SETS APPLIED TO *BCK/BCI*-ALGEBRAS

YOUNG BAE JUN

Department of Mathematics Education
Gyeongsang National University, Jinju 52828, Korea

e-mail: skywine@gmail.com

SEOK-ZUN SONG¹

Department of Mathematics
Jeju National University, Jeju 63243, Korea

e-mail: szsong@jejunu.ac.kr

AND

EUN HWAN ROH

Department of Mathematics Education
Chinju National University of Education, Jinju 52673, Korea

e-mail: ehroh9988@gmail.com

Abstract

The concept of a (strong) set-valued *BCK/BCI*-morphism in *BCK/BCI*-algebras is considered, and several properties are investigated. Conditions for a set-valued mapping to be a set-valued *BCK/BCI*-morphism are given. Using the concept of generalized approximation space, generalized rough subalgebra (ideal) in *BCK/BCI*-algebras are introduced, and investigate their properties. Using the concept of generalized approximation space and ideal of *BCK/BCI*-algebra, another type of generalized lower and upper approximations based on the ideal is considered, and then several properties are investigated.

Keywords: generalized approximation space, generalized rough set, (strong) set-valued *BCK/BCI*-morphism, generalized lower rough subalgebra (ideal), generalized upper rough subalgebra (ideal).

2010 Mathematics Subject Classification: 06F35, 03G25.

¹Corresponding author.

This research is financially supported by the 2021 scientific promotion program funded by Jeju National University.

REFERENCES

- [1] B. Davvaz, *Roughness in rings*, Inform. Sci. **164** (2004) 147–163.
<https://doi.org/10.1016/j.ins.2003.10.001>
- [2] B. Davvaz, *A short note on algebraic T-rough sets*, Inform. Sci. **178** (2008) 3247–3252.
<https://doi.org/10.1016/j.ins.2008.03.014>
- [3] B. Davvaz and M. Mahdavi-pour, *Rough approximations in a general approximation space and their fundamental properties*, Int. J. Gen. Syst. **37** (2008), 373–386.
<https://doi.org/10.1080/03081070701250994>
- [4] W.A. Dudek, Y.B. Jun and H.S. Kim, *Rough set theory applied to BCI-algebras*, Quasigroups and Related Systems **9** (2002) 45–54.
- [5] Y. Huang, *BCI-algebra* (Science Press, Beijing, China, 2006).
- [6] K. Iséki, *On BCI-algebras*, Math. Seminar Notes **8** (1980) 125–130.
- [7] K. Iséki and S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japon. **23** (1978) 1–26.
- [8] M. Kondo, *On the structure of generalized rough sets*, Inform. Sci. **176** (2006) 589–600.
<https://doi.org/10.1016/j.ins.2005.01.001>
- [9] M. Kryszkiewicz, *Rough set approach to incomplete information systems*, Inform. Sci. **112** (1998) 39–49. [https://doi.org/10.1016/S0020-0255\(98\)10019-1](https://doi.org/10.1016/S0020-0255(98)10019-1)
- [10] N. Kuroki, *Rough ideals in semigroups*, Inform. Sci. **100** (1997) 139–163.
[https://doi.org/10.1016/S0020-0255\(96\)00274-5](https://doi.org/10.1016/S0020-0255(96)00274-5)
- [11] V. Leoreanu-Fotea and B. Davvaz, *Roughness in n-ary hypergroups*, Inform. Sci. **178** (2008) 4114–4124.
<https://doi.org/10.1016/j.ins.2008.06.019>
- [12] J. Liang, J. Wang and Y. Qian, *A new measure of uncertainty based on knowledge granulation for rough sets*, Inform. Sci. **179** (2009) 45–470.
<https://doi.org/10.1016/j.ins.2008.10.010>
- [13] J. Meng and Y.B. Jun, *BCK-Algebras*, Kyungmoon Sa Co. (Seoul, Korea, 1994).
- [14] Z. Meng and Z. Shi, *A fast approach to attribute reduction in incomplete decision systems with tolerance relation-based rough sets*, Inform. Sci. **179** (2009) 2774–2793.
<https://doi.org/10.1016/j.ins.2009.04.002>
- [15] D.Q. Miao, Y. Zhao, Y.Y. Yao, H.X. Li and F.F. Xu, *Relative reducts in consistent and inconsistent decision tables of the Pawlak rough set model*, Inform. Sci. **179** (2009) 4140–4150.
<https://doi.org/10.1016/j.ins.2009.08.020>
- [16] J.N. Mordeson, *Rough set theory applied to (fuzzy) ideal theory*, Fuzzy Sets and Systems **121** (2001) 315–324.
[https://doi.org/10.1016/S0165-0114\(00\)00023-3](https://doi.org/10.1016/S0165-0114(00)00023-3)

- [17] Z. Pawlak, *Rough sets*, Int. J. Inform. Comp. Sci. **11** (1982) 341–356.
<https://doi.org/10.1007/BF01001956>
- [18] Z. Pawlak and A. Skowron, *Rudiments of rough sets*, Inform. Sci. **177** (2007) 3–27.
<https://doi.org/10.1016/j.ins.2006.06.003>
- [19] Z. Pawlak and A. Skowron, *Rough sets: some extensions*, Inform. Sci. **177** (2007) 28–40.
<https://doi.org/10.1016/j.ins.2006.06.006>
- [20] Z. Pawlak and A. Skowron, *Rough sets and boolean reasoning*, Inform. Sci. **177** (2007) 41–73.
<https://doi.org/10.1016/j.ins.2006.06.007>
- [21] D. Pei, *On definable concepts of rough set models*, Inform. Sci. **177** (2007) 4230–4239.
<https://doi.org/10.1016/j.ins.2007.01.020>
- [22] Y. Qian, J. Liang, D. Li, H. Zhang and C. Dang, *Measures for evaluating the decision performance of a decision table in rough set theory*, Inform. Sci. **178** (2008) 181–202.
<https://doi.org/10.1016/j.ins.2007.08.010>
- [23] A. Skowron and J. Stepaniuk, *Generalized approximation spaces*, in: T.Y. Lin, A.M. Wildberger (Eds.), *Soft Computing*, The Society for Computer Simulation (San Diego, 1995) 18–21.
- [24] J. Stepaniuk, *Approximation spaces in extensions of rough set theory*, in: L. Polkowski, A. Skowron (Eds.), *RSCTC98*, LNAI **1424** (1998) 290–297.
https://doi.org/10.1007/3-540-69115-4_40
- [25] Q. Wang and J. Zhan, *Rough semigroups and rough fuzzy semigroups based on fuzzy ideals*, Open Math. **14** (2016) 1114–1121.
<https://doi.org/10.1515/math-2016-0102>
- [26] W.Z. Wu and W.X. Zhang, *Constructive and axiomatic approaches of fuzzy approximation operators*, Inform. Sci. **159** (2004) 233–254.
<https://doi.org/10.1016/j.ins.2003.08.005>
- [27] W.Z. Wu, J.S. Mi and W.X. Zhang, *Generalized fuzzy rough sets*, Inform. Sci. **151** (2003) 263–282.
[https://doi.org/10.1016/S0020-0255\(02\)00379-1](https://doi.org/10.1016/S0020-0255(02)00379-1)
- [28] [54] W.Z. Wu, Y. Leung and J.-S. Mi, *On characterizations of $(\mathcal{I}, \mathcal{T})$ -fuzzy rough approximation operators*, Fuzzy Sets and Systems **154** (2005) 76–102.
<https://doi.org/10.1016/j.fss.2005.02.011>
- [29] W.-Z. Wu and W.-X. Zhang, *Neighborhood operator systems and approximations*, Inform. Sci. **144** (2002) 201–217.
[https://doi.org/10.1016/S0020-0255\(02\)00180-9](https://doi.org/10.1016/S0020-0255(02)00180-9)
- [30] U. Wybraniec-Skardowska, *On a generalization of approximation space*, Bull. Polish Acad. Sci. Math. **37** (1989) 51–65.
- [31] O.G. Xi, *Fuzzy BCK-algebra*, Math. Japon. **36** (1991) 935–942.

- [32] Q.-M. Xiao and Z.-L. Zhang, *Rough prime ideals and rough fuzzy prime ideals in semigroups*, Inform. Sci. **176** (2006) 725–733.
<https://doi.org/10.1016/j.ins.2004.12.010>
- [33] S. Yamak, O. Kazanci and B. Davvaz, *Generalized lower and upper approximations in a ring*, Inform. Sci. **180** (2010) 1759–1768.
<https://doi.org/10.1016/j.ins.2009.12.026>
- [34] Y. Yao, *Three-way decisions with probabilistic rough sets*, Inform. Sci. **180** (2010) 341–353.
<https://doi.org/10.1016/j.ins.2009.09.021>
- [35] Y.Y. Yao and T.Y. Lin, *Generalization of rough sets using modal logic*, Intel. Automat. Soft Comp., Int. J. **2** (1996) 103–120.
<https://doi.org/10.1080/10798587.1996.10750660>
- [36] Y. Yao, *On generalizing Pawlak approximation operators*, LNAI **1424** (1998) 298–307.
https://doi.org/10.1007/3-540-69115-4_41
- [37] Y.Y. Yao, *Constructive and algebraic methods of the theory of rough sets*, Inform. Sci. **109** (1998) 2–47.
[https://doi.org/10.1016/S0020-0255\(98\)00012-7](https://doi.org/10.1016/S0020-0255(98)00012-7)
- [38] L.A. Zadeh, *Toward a generalized theory of uncertainty (GTU) – an outline*, Inform. Sci. **172** (2005) 1–40.
<https://doi.org/10.1109/GRC.2005.1547227>
- [39] W. Zhu, *Relationship between generalized rough sets based on binary relation and covering*, Inform. Sci. **179** (2009) 210–225.
<https://doi.org/10.1016/j.ins.2008.09.015>
- [40] W. Zhu, *Generalized rough sets based on relations*, Inform. Sci. **177** (22) (2007) 4997–5011.
<https://doi.org/10.1016/j.ins.2007.05.037>
- [41] W. Zhu and Fei-Yue Wang, *On three types of covering rough sets*, IEEE Transactions On Knowledge and Data Engineering **19** (8) (2007) 1131–1144.
<https://doi.org/10.1109/TKDE.2007.1044>
- [42] W. Zhu, *Topological approaches to covering rough sets*, Inform. Sci. **177** (2007) 1499–1508.
<https://doi.org/10.1016/j.ins.2006.06.009>
- [43] W. Zhu, *Relationship among basic concepts in covering-based rough sets*, Inform. Sci. **179** (2009) 2478–2486.
<https://doi.org/10.1016/j.ins.2009.02.013>

Received 24 March 2019
First Revised 27 May 2019
Second Revised 3 June 2019
Accepted 23 September 2020