

## ON HOM-LEIBNIZ AND HOM-LIE-YAMAGUTI SUPERALGEBRAS

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### Abstract

In this paper some characterizations of Hom-Leibniz superalgebras are given and some of their basic properties are found. These properties can be seen as a generalization of corresponding well-known properties of Hom-Leibniz algebras. Considering the Hom-Akivis superalgebra associated to a given Hom-Leibniz superalgebra, it is observed that the Hom-super Akivis identity leads to an additional property of Hom-Leibniz superalgebras, which in turn gives a necessary and sufficient condition for Hom-super Lie admissibility of Hom-Leibniz superalgebras. We also show that every (left) Hom-Leibniz superalgebra has a natural super Hom-Lie-Yamaguti structure.

**Keywords:** Hom-Leibniz superalgebras, Hom-Akivis superalgebras, Hom-Lie-Yamaguti superalgebras.

**2010 Mathematics Subject Classification:** 17A30, 17A32, 17D99.

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Received 19 May 2020  
Revised 3 August 2020  
Accepted 3 August 2020