

## ON HOM-LEIBNIZ AND HOM-LIE-YAMAGUTI SUPERALGEBRAS

SYLVAIN ATTAN<sup>1</sup>, DONATIEN GAPARAYI<sup>2</sup>

AND

A. NOUROU ISSA<sup>1</sup>

<sup>1</sup>*Département de Mathématiques*  
*Université d'Abomey-calavi*  
*01 BP 4521, Cotonou 01, Bénin*

<sup>2</sup>*Ecole Normale Supérieure (E.N.S),*  
*BP 6983 Bujumbura, Burundi*

**e-mail:** sylvane2010@yahoo.fr  
gapadona@yahoo.fr  
woraniss@yahoo.fr

### Abstract

In this paper some characterizations of Hom-Leibniz superalgebras are given and some of their basic properties are found. These properties can be seen as a generalization of corresponding well-known properties of Hom-Leibniz algebras. Considering the Hom-Akivis superalgebra associated to a given Hom-Leibniz superalgebra, it is observed that the Hom-super Akivis identity leads to an additional property of Hom-Leibniz superalgebras, which in turn gives a necessary and sufficient condition for Hom-super Lie admissibility of Hom-Leibniz superalgebras. We also show that every (left) Hom-Leibniz superalgebra has a natural super Hom-Lie-Yamaguti structure.

**Keywords:** Hom-Leibniz superalgebras, Hom-Akivis superalgebras, Hom-Lie-Yamaguti superalgebras.

**2010 Mathematics Subject Classification:** 17A30, 17A32, 17D99.

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Received 19 May 2020  
Revised 3 August 2020  
Accepted 3 August 2020