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APPLYING THE CZÉDLI-SCHMIDT SEQUENCES TO CONGRUENCE PROPERTIES OF PLANAR SEMIMODULAR LATTICES

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Abstract

Following Grätzer and Knapp, 2009, a planar semimodular lattice L is *rectangular*, if the left boundary chain has exactly one doubly-irreducible element, c_l, and the right boundary chain has exactly one doubly-irreducible element, c_r, and these elements are complementary.

The Czédli-Schmidt Sequences, introduced in 2012, construct rectangular lattices. We use them to prove some structure theorems. In particular, we prove that for a slim (no M_3 sublattice) rectangular lattice L, the congruence lattice $\operatorname{Con} L$ has exactly length$[c_l, 1]$ + length$[c_r, 1]$ dual atoms and a dual atom in $\operatorname{Con} L$ is a congruence with exactly two classes. We also describe the prime ideals in a slim rectangular lattice.

Keywords: lattice, congruence, semimodular, planar, slim.

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