

## JOIN IRREDUCIBLE 2-TESTABLE SEMIGROUPS

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*Dedicated to the 65th birthday of Professor Jorge Almeida.*

### Abstract

A nontrivial pseudovariety is *join irreducible* if whenever it is contained in the complete join of some collection of pseudovarieties, then it is contained in one of the pseudovarieties. A finite semigroup is *join irreducible* if it generates a join irreducible pseudovariety. The present article is concerned with semigroups that are *2-testable* in the sense that they satisfy any equation formed by a pair of words that begin with the same variable, end with the same variable, and share the same set of factors of length two. The main objective is to show that there exist precisely seven join irreducible pseudovarieties of 2-testable semigroups. As a consequence, it is decidable in quadratic time if a finite 2-testable semigroup is join irreducible.

**Keywords:** semigroup, 2-testable, pseudovariety, join irreducible.

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