

JOIN IRREDUCIBLE 2-TESTABLE SEMIGROUPS

EDMOND W.H. LEE

Department of Mathematics
Nova Southeastern University, FL 33314, USA

e-mail: edmond.lee@nova.edu

Dedicated to the 65th birthday of Professor Jorge Almeida.

Abstract

A nontrivial pseudovariety is *join irreducible* if whenever it is contained in the complete join of some collection of pseudovarieties, then it is contained in one of the pseudovarieties. A finite semigroup is *join irreducible* if it generates a join irreducible pseudovariety. The present article is concerned with semigroups that are *2-testable* in the sense that they satisfy any equation formed by a pair of words that begin with the same variable, end with the same variable, and share the same set of factors of length two. The main objective is to show that there exist precisely seven join irreducible pseudovarieties of 2-testable semigroups. As a consequence, it is decidable in quadratic time if a finite 2-testable semigroup is join irreducible.

Keywords: semigroup, 2-testable, pseudovariety, join irreducible.

2010 Mathematics Subject Classification: 20M07.

REFERENCES

- [1] P. Aglianò and J.B. Nation, *Lattices of pseudovarieties*, J. Austral. Math. Soc. Ser. A **46** (1989) 177–183.
doi:10.1017/S1446788700030640
- [2] J. Almeida, *Finite Semigroups and Universal Algebra* (World Scientific, Singapore, 1994).
doi:10.1142/2481
- [3] S. Burris and H.P. Sankappanavar, *A Course in Universal Algebra* (Springer Verlag, New York, 1981).
- [4] A. Escada, *The \mathbf{G} -exponent of a pseudovariety of semigroups*, J. Algebra **223** (2000) 15–36.
doi:10.1006/jabr.1999.7993

- [5] T. Evans, *The lattice of semigroup varieties*, Semigroup Forum **2** (1971) 1–43.
doi:10.1007/BF02572269
- [6] M. Jackson, *Finite semigroups whose variety has uncountably many subvarieties*, J. Algebra **228** (2000) 512–535.
doi:10.1006/jabr.1999.8280
- [7] M. Jackson, *Finiteness properties of varieties and the restriction to finite algebras*, Semigroup Forum **70** (2005) 159–187.
doi:10.1007/s00233-004-0161-x
- [8] M. Jackson and E.W.H. Lee, *Monoid varieties with extreme properties*, Tran. Amer. Math. Soc. **370** (2018) 4785–4812.
doi:10.1090/tran/7091
- [9] E.W.H. Lee, *Identity bases for some non-exact varieties*, Semigroup Forum **68** (2004) 445–457.
doi:10.1007/s00233-003-0029-5
- [10] E.W.H. Lee, *On a simpler basis for the pseudovariety **EDS***, Semigroup Forum **75** (2007) 477–479.
doi:10.1007/s00233-006-0682-6
- [11] E.W.H. Lee, *Combinatorial Rees–Sushkevich varieties are finitely based*, Internat. J. Algebra Comput. **18** (2008) 957–978.
doi:10.1142/S0218196708004755
- [12] E.W.H. Lee, *On the variety generated by some monoid of order five*, Acta Sci. Math. (Szeged) **74** (2008) 509–537.
- [13] E.W.H. Lee, *Combinatorial Rees–Sushkevich varieties that are Cross, finitely generated, or small*, Bull. Aust. Math. Soc. **81** (2010) 64–84.
doi:10.1017/S0004972709000616
- [14] E.W.H. Lee, *Finite basis problem for 2-testable monoids*, Cent. Eur. J. Math. **9** (2011) 1–22.
doi:10.2478/s11533-010-0080-x
- [15] E.W.H. Lee, *Varieties generated by 2-testable monoids*, Studia Sci. Math. Hungar. **49** (2012) 366–389.
doi:10.1556/SScMath.49.2012.3.1211
- [16] E.W.H. Lee, J. Rhodes and B. Steinberg, *Join irreducible semigroups*, Internat. J. Algebra Comput. **29** (2019) 1249–1310.
doi:10.1142/S0218196719500498
- [17] E.W.H. Lee and M.V. Volkov, *On the structure of the lattice of combinatorial Rees–Sushkevich varieties*, in: Semigroups and Formal Languages (Lisbon 2005), André, Fernandes, Branco, Gomes, Fountain and Meakin (Ed(s)), (World Scientific, Singapore, 2007) 164–187.
doi:10.1142/9789812708700_0012

- [18] E.W.H. Lee and M.V. Volkov, *Limit varieties generated by completely 0-simple semigroups*, Internat. J. Algebra Comput. **21** (2011) 257–294.
doi:10.1142/S0218196711006169
- [19] S.A. Malyshev, *Permutational varieties of semigroups whose lattice of subvarieties is finite* (in Russian), in: Modern Algebra, (Leningrad. Univ., Leningrad, 1981) 71–76.
- [20] J. Rhodes and B. Steinberg, *The q -theory of Finite Semigroups*, Springer Monographs in Mathematics (Springer, Berlin, 2009).
doi:10.1007/b104443
- [21] M.V. Sapir, *Problems of Burnside type and the finite basis property in varieties of semigroups*, Math. USSR-Izv. **30** (1988) 295–314. [Translation of Izv. Akad. Nauk SSSR Ser. Mat. **51** (1987) 319–340.]
doi:10.1070/IM1988v030n02ABEH001012
- [22] A.N. Trahtman, *Some finite infinitely basable semigroups* (in Russian), Ural. Gos. Univ. Mat. Zap **14** (1987) 128–131.
- [23] A.N. Trahtman, *A six-element semigroup that generates a variety with a continuum of subvarieties* (in Russian), Ural. Gos. Univ. Mat. Zap **14** (1988) 138–143.
- [24] A.N. Trahtman, *Identities of a five-element 0-simple semigroup*, Semigroup Forum **48** (1994) 385–387.
doi:10.1007/BF02573687
- [25] A.N. Trahtman, *Identities of locally testable semigroups*, Comm. Algebra **27** (1999) 5405–5412.
doi:10.1080/00927879908826762
- [26] B.M. Vernikov and M.V. Volkov, *Lattices of nilpotent varieties of semigroups. II* (in Russian), Izv. Ural. Gos. Univ. Mat. Mekh. **1** (10) (1998) 13–33.
- [27] M.V. Volkov, *The finite basis question for varieties of semigroups*, Math. Notes **45** (3) (1989) 187–194. [Translation of Mat. Zametki **45** (3) (1989) 12–23.]
doi:10.1007/BF01158553

Received 4 December 2019

Revised 3 July 2020

Accepted 3 July 2020