

SUMS OF POWERS AND SPECIAL POLYNOMIALS

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Abstract

In this paper, we discuss sums of powers $1^p + 2^p + \cdots + n^p$ and compute both the exponential and ordinary generating functions for these sums. We express these generating functions in terms of exponential and geometric polynomials and also show their connection to other interesting series. In particular, we show their connection to an interesting problem of Ovidiu Furdui.

Keywords: sum of powers, exponential polynomial, geometric polynomial, generating function, harmonic numbers, exponential integral function.

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