Discussiones Mathematicae
General Algebra and Applications 40 (2020) 165–167
doi:10.7151/dmgaa.1333

COMMUTATIVITY WITH DERIVATIONS OF SEMIPRIME RINGS

Mehsin Jabel Atteya

Department of Mathematics, University of Leicester
University Road, Leicester LE1 7RH, United Kingdom

Department of Mathematics, College of Education
Al-Mustansiriyah University, Iraq

e-mail: mjaas2@leicester.ac.uk

Abstract

Let R be a 2-torsion free semiprime ring with the centre $Z(R)$, U be a non-zero ideal and $d\colon R \to R$ be a derivation mapping. Suppose that R admits

(1) a derivation d satisfying one of the following conditions:
  (i) $[d(x), d(y)] - [x, y] \in Z(R)$ for all $x, y \in U$,
  (ii) $[d^2(x), d^2(y)] - [x, y] \in Z(R)$ for all $x, y \in U$,
  (iii) $[d(x)^2, d(y)^2] - [x, y] \in Z(R)$ for all $x, y \in U$,
  (iv) $[d(x^2), d(y^2)] - [x, y] \in Z(R)$ for all $x, y \in U$,
  (v) $[d(x), d(y)] - [x^2, y^2] \in Z(R)$ for all $x, y \in U$.

(2) a non-zero derivation d satisfying one of the following conditions:
  (i) $d([d(x), d(y)]) - [x, y] \in Z(R)$ for all $x, y \in U$,
  (ii) $d([d(x), d(y)]) + [x, y] \in Z(R)$ for all $x, y \in U$.

Then R contains a non-zero central ideal.

Keywords: semiprime rings, derivations, torsion free rings, central ideal.

2010 Mathematics Subject Classification: 16W25, 16N60, 16U80.

References

[1] M.A. Chaudhry and Allah-Bakhsh Thaheem, *On a pair of (α, β)-derivation of semiprime rings*, Aequation Math. **69** (2005) 224–230. doi:10.1007/s00010-004-2763-5

[2] H.E. Bell and W.S. Martindal III, *Centralizing mappings of semiprime rings*, Canad. Math. Bull. 30 **10** (1987) 92–101. doi:10.4153/CMB-1987-014-x

[3] M. Brešar, *On the distance of the compositions of two derivations to the generalized derivations*, Glasgow Math. J. **33** (1991) 80–93.

[4] M.N. Daif and H.E. Bell, *Remarks on derivations on semiprime rings*, Int. J. Math. and Math. Sci. **15** (1992) 205–206.
doi:10.1155/S0161171292000255

[5] M.N. Daif, *Commutativity results for semiprime rings with derivations*, Int. J. Math. and Math. Sci. **21** (1998) 471–474.
doi:10.1155/S0161171298000660

[6] V. De Filippis, *Automorphisms and derivations in prime rings*, Rcndiconti di Mathematica, Serie VII, **19** (1999) 393–404.

[7] I.N. Herstein, Topics in Ring Theory (University of Chicago Press, Chicago, 1969).

[8] C. Lanski, *An Engel condition with derivation for left ideals*, Proc. Amer. Math. Soc. **125** (1997) 339–345.
doi:10.1090/S0002-9939-97-03673-3

[9] A.H. Majeed and Mehsin Jabel, *Some results of prime and semiprime rings with derivations*, Um-Salama Sci. J. **3** (2005) 508–516.

[10] Mehsin Jabel and A.H. Majeed, *On derivations of semiprime rings*, Antarctica J. Math. **4** (2007) 131–138.

[11] M.J. Atteya, *On generalized derivations of semiprime rings*, Int. J. Alg. **12** (2010) 591–598.

[12] A. Nakajima, *On categorical properties of generalized derivations*, Sci. Math. **2** (1999) 345–352.

[13] L. Oukhtite and S. Salhi, *On derivations in σ-prime rings*, Int. J. Alg. **1** (2007) 241–246.
doi:10.12988/ija.2007.07025

[14] B. Zalar, *On centralizers of semiprime rings*, Comment. Math. Univ. Carolinae **32** (1991) 609–614.

[15] I.N. Herstein, Rings with Involution (University of Chicago Press, Chicago, Illinois, 1976).

[16] M. Brešar, *Centralizing mappings and derivations in prime rings*, J. Alg. **156** (1993) 385–394.

[17] X. Xu, Y. Liu and W. Zhang, *Skew N-derivations on semiprime rings*, Bull. Korean Math. Soc. **50** (2013) 1863–1871.
doi:10.4134/BKMS.2013.50.6.1863

[18] P.M. Cohn, Further Algebra and Applications, 1st edition (Springer-Verlag, London, Berlin, Heidelberg, 2003).
doi:10.1007/978-1-4471-0039-3

[19] I.N. Herstein, *A commutativity theorem*, J. Alg. **38** (1976) 112–118.
doi:10.1016/0021-8693(76)90248-9

[20] E.C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957) 1093–1100.
doi:10.2307/2032686

Received 20 August 2019
Revised 27 November 2019
Accepted 18 January 2020