

ANALYTIC PROPERTIES OF THE APOSTOL-VU MULTIPLE FIBONACCI ZETA FUNCTIONS

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Abstract

In this note we study the analytic continuation of the Apostol-Vu multiple Fibonacci zeta functions

$$\zeta_{AVF,k}(s_1, \dots, s_k; s_{k+1}) = \sum_{1 \leq m_1 < \dots < m_k} \frac{1}{F_{m_1}^{s_1} F_{m_2}^{s_2} \cdots F_{m_k}^{s_k} F_{m_1+m_2+\dots+m_k}^{s_{k+1}}},$$

where s_1, \dots, s_{k+1} are complex variables and F_n is the n -th Fibonacci number. We find a complete list of poles and their corresponding residues.

Keywords: analytic continuation, Fibonacci numbers, multiple Fibonacci zeta function, Apostol-Vu multiple Fibonacci zeta function.

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