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STRONGLY GENERALIZED RADICAL SUPPLEMENTED MODULES

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Abstract

We introduce and study strongly generalized radical-supplemented modules (or briefly sgrs-modules). With the notation $Rad_g(R) := \cap\{K : K \leq R_R, K \text{ is both essential and maximal}\}$, we prove that (under some mild conditions on a ring R) every right R-module is a sgrs-module if and only if $\frac{R}{Soc(R)}$ is right perfect and the idempotents lift module $Rad_g(R)$.

Keywords: essential submodules, supplemented modules, strongly radical-supplemented modules, (semi-) perfect rings.

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