

## ROOT SELECTIONS AND $2^p$ -th ROOT SELECTIONS IN HYPERFIELDS

PAWEŁ GŁADKI

*Institute of Mathematics*  
*University of Silesia*  
*ul. Bankowa 14, 40-007 Katowice, Poland*

**e-mail:** pawel.gladki@us.edu.pl

### Abstract

In this paper we define root selections and  $2^p$ -th root selections for hyperfields: these are multiplicative subgroups whose existence is equivalent to the existence of a well behaved square root function and  $2^p$ -th root function, respectively. We proceed to investigate some basic properties of such root selections, and draw some parallels between the theory of root selections for hyperfields and the theory of orderings and orderings of higher level in hyperfields previously studied by the author.

**Keywords:** square roots,  $2^p$ -th roots, orderings, orderings of higher level, hyperfields.

**2010 Mathematics Subject Classification:** 12F05, 12J15.

### REFERENCES

- [1] E. Artin and O. Schreier, *Algebraische Konstruktion reeller Körper*, Abh. Math. Sem. Univ. Hamburg **5** (1927) 85–99.  
doi:10.1007/BF02952512
- [2] E. Artin and O. Schreier, *Eine Kennzeichnung der reell abgeschlossenen Körper*, Abh. Math. Sem. Univ. Hamburg **5** (1927) 225–231.  
doi:10.1007/BF02952522
- [3] E. Becker, *Hereditarily Pythagorean fields and orderings of higher level*, volume 29 of *Monografias de Matemática* (IMPA, Rio de Janeiro, 1978).
- [4] P. Gładki,  *$N$ -th root selections in fields*, to appear.
- [5] P. Gładki, *Orderings of higher level in multirings and multirings*, Ann. Math. Sil. **24** (2010) 15–25.

- [6] P. Gładki and M. Marshall, *Orderings and signatures of higher level on multirings and hyperfields*, *J. K-Theory* **10** (2012) 489–518.  
doi:10.1017/is012004021jkt189
- [7] P. Gładki and M. Marshall, *Witt equivalence of function fields over global fields*, *Trans. Amer. Math. Soc.* **369** (2017) 7861–7881.  
doi:10.1090/tran/6898
- [8] J. Jun, *Algebraic geometry over hyperfields*, *Adv. Math.* **323** (2018) 142–192.  
doi:10.1016/j.aim.2017.10.043
- [9] M. Krasner, *Approximation des corps valués complets de caractéristique  $p \neq 0$  par ceux de caractéristique 0*, in: *Colloque d’algebre supérieure, tenu a Bruxelles du 19 au 22 décembre 1956*, Centre Belge de Recherches Mathématiques, pages 129–206. Établissements Ceuterick, Louvain (Librairie Gauthier-Villars, Paris, 1957).
- [10] T.-Y. Lam, *The theory of ordered fields*, in: B.R. McDonald, editor, *Ring Theory and Algebra III*, volume 55 of *Lecture Notes in Pure and Appl. Math.*, pages 1–152, (Dekker, New York, 1980).
- [11] M. Marshall, *Real reduced multirings and multifields*, *J. Pure Appl. Algebra* **205** (2006) 452–468.  
doi:10.1016/j.jpaa.2005.07.011
- [12] W. Waterhouse, *Square root as homomorphism*, *American Math. Monthly* **119** (2012) 235–239.  
doi:10.4169/amer.math.monthly.119.03.235

Received 5 May 2018

Revised 15 July 2018

Accepted 10 January 2019