

ROOT SELECTIONS AND 2^p -th ROOT SELECTIONS IN HYPERFIELDS

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Abstract

In this paper we define root selections and 2^p -th root selections for hyperfields: these are multiplicative subgroups whose existence is equivalent to the existence of a well behaved square root function and 2^p -th root function, respectively. We proceed to investigate some basic properties of such root selections, and draw some parallels between the theory of root selections for hyperfields and the theory of orderings and orderings of higher level in hyperfields previously studied by the author.

Keywords: square roots, 2^p -th roots, orderings, orderings of higher level, hyperfields.

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