

## ORDERED REGULAR SEMIGROUPS WITH BIGGEST ASSOCIATES

T.S. BLYTH

*Mathematical Institute*  
*University of St Andrews, Scotland*  
**e-mail:** tsb@st-andrews.ac.uk

AND

M.H. ALMEIDA SANTOS

*Centro de Matemática e Aplicações (CMA)*  
*Departamento de Matemática, FCT*  
*Universidade Nova de Lisboa, Portugal*  
**e-mail:** mhas@fct.unl.pt

### Abstract

We investigate the class **BA** of ordered regular semigroups in which each element has a biggest associate  $x^\dagger = \max\{y \mid xyx = x\}$ . This class properly contains the class **PO** of principally ordered regular semigroups (in which there exists  $x^* = \max\{y \mid xyx \leq x\}$ ) and is properly contained in the class **BI** of ordered regular semigroups in which each element has a biggest inverse  $x^\circ$ . We show that several basic properties of the unary operation  $x \mapsto x^*$  in **PO** extend to corresponding properties of the unary operation  $x \mapsto x^\dagger$  in **BA**. We consider naturally ordered semigroups in **BA** and prove that those that are orthodox contain a biggest idempotent. We determine the structure of some such semigroups in terms of a principal left ideal and a principal right ideal. We also characterise the completely simple members of **BA**. Finally, we consider the naturally ordered semigroups in **BA** that do not have a biggest idempotent.

**Keywords:** regular semigroup, biggest associate, principally ordered, naturally ordered.

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