

## THE PLANAR INDEX AND OUTERPLANAR INDEX OF SOME GRAPHS ASSOCIATED TO COMMUTATIVE RINGS

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### Abstract

In this paper, we study the planar and outerplanar indices of some graphs associated to a commutative ring. We give a full characterization of these graphs with respect to their planar and outerplanar indices when  $R$  is a finite ring.

**Keywords:** Jacobson graph, commutative ring, planar index, outerplanar index.

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