

## CONRAD'S PARTIAL ORDER ON P.Q.-BAER \*-RINGS

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### Abstract

We prove that a p.q.-Baer \*-ring forms a pseudo lattice with Conrad's partial order and also characterize p.q.-Baer \*-rings which are lattices. The initial segments of a p.q.-Baer \*-ring with the Conrad's partial order are shown to be an orthomodular posets.

**Keywords:** Conrad's partial order, p.q.-Baer \*-ring, central cover, orthomodular set.

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### REFERENCES

- [1] A. Abian, *Direct product decomposition of commutative semisimple rings*, Proc. Amer. Math. Soc. **24** (1970) 502–507.  
doi:10.2307/2037396
- [2] J.K. Baksalary and S.K. Mitra, *Left-star and right-star partial ordering*, Linear Algebra Appl. **149** (1991) 73–89.  
doi:10.1016/0024-3795(91)90326-R

- [3] S.K. Berberian, *Baer  $*$ -Rings*, Grundlehren Math. Wiss. Band 195. Vol. **296** (Berlin, Springer, 1972).  
doi:10.1007/978-3-642-15071-5
- [4] G.F. Birkenmeier, J.K. Park and S.T. Rizvi, *Principally quasi-Baer rings hulls*, Advances in Ring Theory Trends in Mathematics, 47–61 (Birkhäuser Basel, 2010).  
doi:10.1007/978-3-0346-0286-0\4
- [5] G.F. Birkenmeier, J.K. Park and S.R. Tariq, *Extensions of Rings and Modules* (New York, Birkhäuser, 2013).  
doi:10.1007/978-0-387-92716-9
- [6] W.D. Burgess and R. Raphael, *On Conrad's partial order relation on semiprime rings and on semigroups*, Semigroup Forum **16** (1978) 133–140.  
<http://eudml.org/doc/134282>
- [7] J. Cīrulis, *Quasi-orthomodular posets and weak BCK-algebras*, Order **31** (2014) 403–419.  
doi:10.1007/s11083-013-9309-1
- [8] P.F. Conrad, *The hulls of semiprime rings*, Austral. Math. Soc. **12** (1975) 311–314.  
doi:10.1017/S0004972700023911
- [9] G. Dolinar and J. Marovt, *Star partial order on  $B(H)$* , Linear Algebra Appl. **434** (2011) 319–326.  
doi:10.1016/j.laa.2010.08.023
- [10] G. Dolinar, B. Kuzma and J. Marovt, *A note on partial orders of Hartwig, Mitsch, and Šemrl*, Appl. Math. and Comp. **270** (2015) 711–713.  
doi:10.1016/j.amc.2015.08.066
- [11] M.P. Drazin, *Natural structure on semigroup with involution*, Bull. Amer. Math. Soc. **84** (1978) 139–141.  
<https://projecteuclid.org/euclid.bams/1183540393>
- [12] R.E. Hartwig, *How to partially order regular elements*, Math. Japon. **25** (1980) 1–13.
- [13] R.E. Hartwig, *Pseudo lattice properties of the star-orthogonal partial ordering for star-regular rings.*, Proc. Amer. Math. Soc. **77** (1979) 299–303.  
doi:10.2307/2042174
- [14] C.Y. Hong, N.K. Kim, T.K. Kwak, *Ore extension of Baer and PP rings*, J. Pure Appl. Algebra **151** (2000) 215–226.  
doi:10.1016/S0022-4049(99)00020-1
- [15] M.F. Janowitz, *On the  $*$ -order for Rickart  $*$ -rings*, Algebra Universalis **16** (1983) 360–369.  
doi:10.1007/BF01191791
- [16] I. Kaplansky, *Rings of Operators* (W.A. Benjamin, Inc., New York-Amsterdam, 1968).
- [17] A. Khairnar and B.N. Waphare, *Unitification of weakly  $p.q.$ -Baer  $*$ -rings*, Southeast Asian Bull. Math. (to appear).  
arXiv:1612.01681

- [18] A. Khairnar and B.N. Waphare, *Order properties of generalized projections*, Linear Multilinear Algebra **65** (2017) 1446–1461.  
doi:10.1080/03081087.2016.1242554
- [19] A. Khairnar and B.N. Waphare, *Generalized Projections in  $\mathbb{Z}_n$* , AKCE Int. J. Graphs Comb.  
doi:10.1016/j.akcej.2018.01.010
- [20] A. Khairnar and B.N. Waphare, *A Sheaf Representation of Principally Quasi-Baer \*-Rings*, Algebr. Represent. Theory.  
<https://doi.org/10.1007/s10468-017-9758-0>
- [21] J.Y. Kim, *On reflexive principally quasi-Baer rings*, Korean J. Math. **17** (2009) 233–236.
- [22] I. Krēmere, *Left-star order structure of Rickart \*-ring*, Linear Multilinear Algebra **64** (2016) 341–352.  
doi:10.1080/03081087.2015.1040369
- [23] F.W. Levi, *Ordered groups*, Proc. Indian Acad. Sci. A **16** (1942) 256–263.
- [24] S. Maeda, *On the lattice of projections of a Baer \*-ring*, J. Sci. Hiroshima Univ. Ser. A **22** (1958) 75–88.
- [25] P. Šemrl, *Automorphisms of  $B(H)$  with respect to minus partial order*, J. Math. Anal. Appl. **369** (2010) 205–213.  
doi:10.1016/j.jmaa.2010.02.059
- [26] B.N. Waphare and Anil Khairnar, *Semi-Baer modules*, J. Algebra Appl. **14** (2015) 1550145 (12 pages).  
doi:10.1142/S0219498815501455

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