

## QUADRATIC APPROXIMATION OF GENERALIZED TRIBONACCI SEQUENCES

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### Abstract

In this paper, we give quadratic approximation of generalized Tribonacci sequence  $\{V_n\}_{n \geq 0}$  defined by  $V_n = rV_{n-1} + sV_{n-2} + tV_{n-3}$  ( $n \geq 3$ ) and use this result to give the matrix form of the  $n$ -th power of a companion matrix of  $\{V_n\}_{n \geq 0}$ . Then we re-prove the cubic identity or Cassini-type formula for  $\{V_n\}_{n \geq 0}$  and the Binet's formula of the generalized Tribonacci quaternions.

**Keywords:** Binet's formula, companion matrix, generalized Tribonacci sequence, Narayana number, Padovan number, quadratic approximation, Tribonacci number.

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