

IDEMPOTENT ELEMENTS OF WEAK PROJECTION GENERALIZED HYPERSUBSTITUTIONS

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Abstract

A generalized hypersubstitution of type $\tau = (n_i)_{i \in I}$ is a mapping σ which maps every operation symbol f_i to the term $\sigma(f_i)$ and may not preserve arity. It is the main tool to study strong hyperidentities that are used to classify varieties into collections called strong hypervarieties. Each generalized hypersubstitution can be extended to a mapping $\hat{\sigma}$ on the set of all terms of type τ . A binary operation on $\text{Hyp}_G(\tau)$, the set of all generalized hypersubstitutions of type τ , can be defined by using this extension. The set $\text{Hyp}_G(\tau)$ together with such a binary operation forms a monoid, where a hypersubstitution σ_{id} , which maps f_i to $f_i(x_1, \dots, x_{n_i})$ for every $i \in I$, is the neutral element of this monoid. A weak projection generalized hypersubstitution of type τ is a generalized hypersubstitution of type τ which maps at least one of the operation symbols to a variable. In semigroup theory, the various types of its elements are widely considered. In this paper, we present the characterizations of idempotent weak projection generalized hypersubstitutions of type (m, n) and give some sufficient conditions for a weak projection generalized hypersubstitution of type (m, n) to be regular, where $m, n \geq 1$.

Keywords: idempotent, regular, generalized hypersubstitution, weak projection generalized hypersubstitution.

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