Discussiones Mathematicae
General Algebra and Applications 38 (2018) 79–80
doi:10.7151/dmgaa.1278

Full PDF

DM-GAA Page

GENERALIZED CHEBYSHEV POLYNOMIALS

Mourad Abchiche

and

Hacéne Belbachir

Faculty of Mathematics
USTHB, RECITS Laboratory
BP 32, El Alia, 16111 Bab Ezzouar, Algiers, Algeria

e-mail: hbelbachir@usthb.dz
mabchiche@usthb.dz

Abstract

Let $h(x)$ be a non constant polynomial with rational coefficients. Our aim is to introduce the $h(x)$-Chebyshev polynomials of the first and second kind T_n and U_n. We show that they are in a $\mathbb{Q}$-vectorial subspace $E_n(x)$ of $\mathbb{Q}[x]$ of dimension n. We establish that the polynomial sequences $(h^k T_{n-k})_k$ and $(h^k U_{n-k})_k$, $(0 \leq k \leq n-1)$ are two bases of $\mathbb{E}_n(x)$ for which T_n and U_n admit remarkable integer coordinates.

Keywords: Chebyshev polynomials, integer coordinates, polynomial bases.
2010 Mathematics Subject Classification: 11B83, 15A03, 11B39, 08A40.

References

[1] P. Appell and J. Kampé de Fériet, Fonctions Hypergéométriques et Hypersphériques. Polynomes d'Hermite, Gauthier-Villars (Paris, 1926).

[2] H. Belbachir and F. Bencherif, *On some properties of Chebychev polynomials*, Discuss. Math. General Alg. and Appl. **28** (2008) 121–133. doi:10.7151/dmgaa.1138

[3] H. Belbachir and F. Bencherif, *Linear recurrent sequences and powers of a square matrix*, Integers **6** (2006), A12, 17pp.

[4] H. Belbachir and F. Bencherif, *On some properties of Bivariate Fibonacci and Lucas Polynomials*, J. Integer Sequences **11** (2008) 08.2.6.

[5] H. Belbachir and A. Benmezai, *Expansion of Fibonacci and Lucas polynomials: An Answer to Prodinger's question*, J. Integer Sequences, Article 12.7.6, 2012.

[6] C. Cesarano, *Integral representations and new generating functions of Chebyshev polynomials*, Hacet. J. Math. Stat. **44** (2015) 535–546. doi:10.15672/hjms.20154610029

[7] C. Cesarano, *Generalized Chebyshev polynomials*, Hacet. J. Math. Stat. **43** (2014) 731–740.

[8] C. Cesarano, *Identities and generating functions on Chebyshev polynomials*, Georgian Math. J. **19** (2012) 427–440. doi:10.1515/gmj-2012-0031

[9] E. Lucas, Théorie des Nombres (Gautier-Villars, Paris, 1891).

[10] H. Prodinger, *On the expansion of Fibonacci and Lucas polynomials*, J. Integer Sequences **12** (2009), Article 09.1.6.

[11] T.J. Rivlin, Chebychev Polynomials: from Approximation Theory to Algebra and Number Theory, Second Edition (Wiley interscience, 1990).

Received 15 January 2018
Revised 22 January 2018
Accepted 19 February 2018