*Discussiones Mathematicae*
*General Algebra and Applications 37* (*2017*) *223–225*
*doi:10.7151/dmgaa.1274*



# A NOTE ON ADDITIVE GROUPS OF SOME SPECIFIC TORSION-FREE RINGS OF RANK THREE AND MIXED ASSOCIATIVE RINGS

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**Abstract**

It is studied how rank two pure subgroups of a torsion-free Abelian group of rank three influences its structure and type set. In particular, the criterion for such a subgroup $B$ to be a direct summand of a torsion-free Abelian group of rank three with the finite type set containing the greatest element which does not belong to the type set of $B$, is presented. Some results for nil groups and the square subgroup of a decomposable torsion-free Abelian group are also achieved. Moreover, new results for mixed Abelian groups supporting only associative rings are obtained. In particular, the first example of an Abelian group supporting only associative rings but not only commutative rings is given.

**Keywords:** rank, type, square subgroup, additive groups of rings.

**2010 Mathematics Subject Classification:** 20K15, 20K21, 20K27.

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Received 22 June 2017
Revised 28 August 2017
Accepted 8 September 2017