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# GENERALIZED DERIVATIONS WITH LEFT ANNIHILATOR CONDITIONS IN PRIME AND SEMIPRIME RINGS

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**Abstract**

Let $R$ be a prime ring with its Utumi ring of quotients $U$, $C = Z(U)$ be the extended centroid of $R$, $H$ and $G$ two generalized derivations of $R$, $L$ a noncentral Lie ideal of $R$, $I$ a nonzero ideal of $R$. The left annihilator of $S \subseteq R$ is denoted by $l_R(S)$ and defined by $l_R(S) = \{x \in R \,|\, xS = 0\}$. Suppose that $S = \{H(u^n)u^n + u^nG(u^n) \,|\, u \in L\}$ and $T = \{H(x^n)x^n + x^nG(x^n) \,|\, x \in I\}$, where $n \geq 1$ is a fixed integer. In the paper, we investigate the cases when the sets $l_R(S)$ and $l_R(T)$ are nonzero.

**Keywords:** prime ring, derivation, Lie ideal, generalized derivation, extended centroid, Utumi quotient ring.

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