

## ON THE SECOND SPECTRUM OF LATTICE MODULES

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### Abstract

The second spectrum  $Spec^s(M)$  is the collection of all second elements of  $M$ . In this paper, we study the topology on  $Spec^s(M)$ , which is a generalization of the Zariski topology on the prime spectrum of lattice modules. Besides some properties,  $Spec^s(M)$  is characterized and the interrelations between the topological properties of  $Spec^s(M)$  and the algebraic properties of  $M$ , are studied.

**Keywords:** second element, prime element, maximal element, minimal element, spectral space.

**2010 Mathematics Subject Classification:** 06D10, 06E10, 06F10.

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Received 31 August 2016

Revised 8 January 2017