Discussiones Mathematicae
General Algebra and Applications 37 (2017) 31–33
doi:10.7151/dmgaa.1265

ZERO-DIVISOR GRAPHS OF REDUCED RICKART $*$-RINGS

A.A. Patil

Department of Mathematics
Garware College of Commerce, Pune-411004, India

e-mail: avipmj@gmail.com

AND

B.N. Waphare

Center for Advanced Studies in Mathematics
Department of Mathematics
Savitribai Phule Pune University, Pune-411007, India

e-mail: waphare@yahoo.com
bnwaph@math.unipune.ac.in

Abstract

For a ring A with an involution $*$, the *zero-divisor graph* of A, $\Gamma^*(A)$, is the graph whose vertices are the nonzero left zero-divisors in A such that distinct vertices x and y are adjacent if and only if $xy^* = 0$. In this paper, we study the zero-divisor graph of a Rickart $*$-ring having no nonzero nilpotent element. The distance, diameter, and cycles of $\Gamma^*(A)$ are characterized in terms of the collection of prime strict ideals of A. In fact, we prove that the clique number of $\Gamma^*(A)$ coincides with the cellularity of the hull-kernel topological space $\Sigma(A)$ of the set of prime strict ideals of A, where cellularity of the topological space is the smallest cardinal number m such that every family of pairwise disjoint non-empty open subsets of the space have cardinality at most m.

Keywords: reduced ring, Rickart $*$-ring, zero-divisor graph, prime strict ideals.

2010 Mathematics Subject Classification: Primary: 16W10, Secondary: 05C25, 05C15.

References

[1] S. Akbari and A. Mohammadian, *On the zero-divisor graph of a commutative ring*, J. Algebra **274** (2004) 847–855. doi:10.1016/S0021-8693(03)00435-6

[2] D.F. Anderson, R. Levy and J. Shapirob, *Zero-divisor graphs, von Neumann regular rings and Boolean algebras*, J. Pure Appl. Alg. **180** (2003) 221–241. doi:10.1016/S0022-4049(02)00250-5

[3] D.F. Anderson and P.S. Livingston, *The zero-divisor graph of a commutative ring*, J. Algebra **217** (1999) 434–447. doi:10.1006/jabr.1998.7840

[4] I. Beck, *Coloring of commutative rings*, J. Algebra **116** (1988) 208–226. doi:10.1016/0021-8693(88)90202-5

[5] S.K. Berberian, Baer $*$-Rings (Springer-Verlag, Berlin and New York, 1972).

[6] D. Dolžan and P. Oblak, *The zero divisor graphs of rings and semirings*, Int. J. Algebra Comput. **22** 1250033 (2012) 20pp. doi:10.1142/S0218196712500336

[7] F. DeMeyer, T. McKenzie and K. Schneider, *The zero-divisor graph of a commutative semigroup*, Semigroup Forum **65** (2002) 206–214. doi:10.1007/s002330010128

[8] F. DeMeyer and L. DeMeyer, *Zero divisor graph of Semigroups*, J. Algebra **283** (2005) 190–198. doi:10.1016/j.jalgebra.2004.08.028

[9] R. Halaš and M. Jukl, *On Beck's colouring of posets*, Discrete Math. **309** (2009) 4584–4589. doi:10.1016/j.disc.2009.02.024

[10] R. Sen Gupta, M.K. Sen and S. Ghosh, *A variation of zero-divisor graphs*, Discuss. Math. Gen. Algebra Appl. **35** (2015) 159–176. doi:10.7151/dmgaa.1238

[11] K. Samei, *The zero divisor graph of a reduced ring*, J. Pure App. Alg. **209** (2007) 813–821. doi:10.1016/j.jpaa.2006.08.008.

[12] S.K. Nimbhorkar, *Coloring of Rickart *-rings*, Trends in Theory of Rings and Modules, S. Tariq Rizvi and S.M.A. Zaidi (Eds.) Anamya Publishers (New Delhi 2005), 91–95.

[13] S.K. Nimbhorkar, M.P. Wasadikar and L. DeMeyer, *Coloring of semilattices*, Ars Comb. **12** (2007) 97–104.

[14] Avinash Patil and B.N. Waphare, *On the Zero-divisor graph of Rickart $*$-rings*, Asian-Eur. J. Math. **10** (2017) 1750015 (17 pages). doi:10.1142/S1793557117500152

[15] Avinash Patil, B.N. Waphare, V.V. Joshi and H.Y. Paurali, *Zero-divisor graphs of lower dismantlable lattices*-I, Math. Slovaca **67** (2017) 1–12.

[16] Avinash Patil, B.N. Waphare and V.V. Joshi, *Zero-divisor graphs of lower dismantlable lattices*-II, to appear in Math. Slovaca.

[17] Avinash Patil and B.N. Waphare, *The zero-divisor graph of a ring with involution*, J. Algebra Appl. (to appear).

[18] S.P. Redmond, *The zero-divisor graph of a non-commutative ring*, Int. J. Commut. Rings **1** (2002) 203–211.

[19] N.K. Thakare and S.K. Nimbhorkar, *Prime strict ideals in Rickart *-rings*, Ind. J. Pure appl. Math. **22** (1991) 63–72.

[20] D.B. West, *Introduction to Graph Theory*, Second Edition (Prentice-Hall of India, New Delhi, 2002).

Received 28 June 2016
1st Revised 17 October 2016
2st Revised 23 January 2017