

ZERO-DIVISOR GRAPHS OF REDUCED RICKART *-RINGS

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Abstract

For a ring A with an involution $*$, the *zero-divisor graph* of A , $\Gamma^*(A)$, is the graph whose vertices are the nonzero left zero-divisors in A such that distinct vertices x and y are adjacent if and only if $xy^* = 0$. In this paper, we study the zero-divisor graph of a Rickart $*$ -ring having no nonzero nilpotent element. The distance, diameter, and cycles of $\Gamma^*(A)$ are characterized in terms of the collection of prime strict ideals of A . In fact, we prove that the clique number of $\Gamma^*(A)$ coincides with the cellularity of the hull-kernel topological space $\Sigma(A)$ of the set of prime strict ideals of A , where cellularity of the topological space is the smallest cardinal number m such that every family of pairwise disjoint non-empty open subsets of the space have cardinality at most m .

Keywords: reduced ring, Rickart $*$ -ring, zero-divisor graph, prime strict ideals.

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