Discussiones Mathematicae
General Algebra and Applications 37 (2017) 45–47
doi:10.7151/dmgaa.1264

SOME HOMOLOGICAL PROPERTIES OF SKEW *PBW* EXTENSIONS ARISING IN NON-COMMUTATIVE ALGEBRAIC GEOMETRY

Oswaldo Lezama

Helbert Venegas
Seminario de Álgebra Constructiva – SAC^2
Departamento de Matemáticas
Universidad Nacional de Colombia, Sede Bogotá

e-mail: jolezamas@unal.edu.co

Abstract

In this short paper we study for the skew *PBW* (Poincar-Birkhoff-Witt) extensions some homological properties arising in non-commutative algebraic geometry, namely, Auslander-Gorenstein regularity, Cohen-Macaulayness and strongly noetherianity. Skew *PBW* extensions include a considerable number of non-commutative rings of polynomial type such that classical *PBW* extensions, quantum polynomial rings, multiplicative analogue of the Weyl algebra, some Sklyanin algebras, operator algebras, diffusion algebras, quadratic algebras in 3 variables, among many others. Parametrization of the point modules of some examples is also presented.

Keywords: Auslander regularity condition, Cohen-Macaulay rings, strongly noetherian algebras, skew *PBW* extensions, filtered-graded rings, point modules.

2010 Mathematics Subject Classification: Primary: 16S38; Secondary: 16W50, 16S80, 16S36.

References

[1] J.P. Acosta, C. Chaparro, O. Lezama, I. Ojeda and C. Venegas, *Ore and Goldie theorems for skew PBW extensions*, Asian-European J. Math. **6** (2013). doi:10.1142/S1793557113500617

[2] J.P. Acosta, O. Lezama and M.A. Reyes, *Prime ideals of skew PBW extensions*, Revista de la Unión Matemtica Argentina **56** (2015) 39–55.

[3] K. Ajitabh, S.P. Smith and J.J. Zhang, *Injective resolutions of some regular rings*, J. Pure Appl. Algebra **140** (1999) 1–21. doi:10.1016/S0022-4049(99)00049-3

[4] V. Artamonov, Quantum Polynomials (WSPC Proceedings, 2008).

[5] V. Artamonov, *Serre's quantum problem*, Russian Math. Surveys **53** (1998) 657–730.
doi:10.1070/RM1998v053n04ABEH000056

[6] M. Artin, L.W. Small and J.J. Zhang, *Generic flatness for strongly Noetherian algebras*, J. Algebra **221** (1999) 579–610.
doi:10.1006/jabr.1999.7997

[7] M. Artin, J. Tate and M. Van den Bergh, *Some algebras associated to automorphisms of elliptic curves*, The Grothendieck Festchrift, Vol. I, Birkhuser Boston (1990) 33–85.
doi:10.1007/978-0-8176-4574-8-3

[8] M. Artin and J.J. Zhang, *Noncommutative projective schemes*, Adv. Mah. **109** (1994) 228–287.
doi:10.1006/aima.1994.1087

[9] A. Bell and K. Goodearl, *Uniform rank over differential operator rings and Poincar-Birkhoff-Witt extensions*, Pacific J. Math. **131** (1988) 13–37.
doi:10.2140/pjm.1988.131.13

[10] J.E. Björk, Rings of Differential Operators, North-Holland Mathematical Library, vol. 21 (North-Holland Publishing Co., Amsterdam-New York, 1979).

[11] J.E. Björk, *The Auslander Condition on Noetherian Rings*, Sm. d'algbre P. Dubreil et M.-P. Malliavin, Lect. Notes Math. **1404** (1989) 137–173.

[12] J. Bueso, J. Gomez-Torrecillas and A. Verschoren, Algorithmic Methods in Noncommutative Algebra: Applications to Quantum Groups (Kluwer, 2003).

[13] J. Gomez-Torrecillas, *Basic Module Theory over Non-Commutative Rings with Computational Aspects of Operator Algebras*, in: M. Barkatou *et al.* (Eds.), AADIOS 2012, Lecture Notes in Computer Science **8372** (2014) 23–82.

[14] J. Gomez-Torrecillas and F.J. Lobillo, *Global homological dimension of Multifiltered rings and quantized enveloping algebras*, J. Algebra **225** (2000) 522–533.
doi:10.1006/jabr.1999.8101

[15] J. Gomez-Torrecillas and F.J. Lobillo, *Auslander-regular and Cohen-Macaulay Quantum groups*, Algebras and Representation Theory **7** (2004) 35–42.
doi:10.1023/B:ALGE.0000019384.36800.fa

[16] E.K. Ekström, *The Auslander condition on graded and filltered noetherian rings*, Sminaire Dubreil-Malliavin 1987–88, Lect. Notes Math. 1404 (Springer Verlag, 1989) 220–245.

[17] G. Krause and T.H. Lenagan, *Growth of algebras and Gelfand-Kirillov dimension*, 2nd ed., Graduate Studies in Mathematics, 22 (AMS, Providence, USA, 2000).

[18] H. Kredel, Solvable Polynomial Rings (Shaker, 1993).

[19] T. Levasseur, *Some properties of non-commutative regular rings*, Glasglow J. Math. **34** (1992) 277–300.
doi:10.1017/S0017089500008843

[20] T. Levasseur and J.T. Stafford, *The quantum coordinate ring of the special linear group*, J. Pure Appl. Algebra **86** (1993) 181–186.
doi:10.1016/0022-4049(93)90102-Y

[21] O. Lezama and C. Gallego, *Gröbner bases for ideals of sigma-PBW extensions*, Communications in Algebra **39** (2011) 50–75.
doi:10.1080/00927870903431209

[22] O. Lezama and M. Reyes, *Some homological properties of skew PBW extensions*, Communications in Algebra **42** (2014) 1200–1230.
doi:10.1080/00927872.2012.735304

[23] O. Lezama and E. Latorre, *Non-commutative algebraic geometry of semi-graded rings*, arXiv:1605.09057 [math.RA].

[24] V. Levandovskyy, Non-Commutative Computer Algebra for Polynomial Algebras: Gröbner Bases, Applications and Implementation (Dissertation, University of Kaiserslautern, 2005).

[25] H. Li, Noncommutative Gröbner Bases and Filtered-Graded Transfer (Springer, 2002).

[26] H. Li, Non-Commutative Zariskian Rings, Ph.D Thesis (Universiteit Antwerpen, Antwerp, Belgium, 1990).

[27] H. Li and F.V. Oystaeyen, *Zariskian filtrations*, Comm. Algebra **17** (1989) 2945–2970.
doi:10.1080/00927878908823888

[28] A. Reyes, *Gelfand-Kirillov dimension of skew PBW extensions*, Revista Colombiana de Matemáticas **47** (2013) 95–111.

[29] A. Reyes, Ring and Module Theoretic Properties of $\sigma - PBW$ Extensions, Tesis de Doctorado (Universidad Nacional de Colombia, Bogotá, 2013).

[30] D. Rogalski, *An introduction to non-commutative projective algebraic geometry*, arXiv:1403.3065 [math.RA].

[31] A. Zaks, *Injective dimension of semiprimary rings*, J. Algebra **13** (1969) 73–89.
doi:10.1016/0021-8693(69)90007-6

Received 19 July 2016
Revised 3 February 2017