

## GENERALIZED PELL EQUATIONS FOR $2 \times 2$ MATRICES

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### Abstract

In this paper we consider the solutions of the generalized matrix Pell equations  $X^2 - dY^2 = cI$ , where  $X$  and  $Y$  are  $2 \times 2$  matrices over  $\mathbb{Z}$ ,  $d$  is a non-zero (positive or negative) square-free integer,  $c$  is an arbitrary integer and  $I$  is the  $2 \times 2$  identity matrix. We determine all solutions of such equations for  $c = \pm 1$ , as well as all non-commutative solutions for an arbitrary  $c$ .

**Keywords:** matrix equations, Pell equation.

**2010 Mathematics Subject Classification:** 15A24, 15B36, 11D09.

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Received 27 January 2016  
1st Revised 29 January 2016  
2st Revised 6 December 2016