

VARIETIES OF REGULAR ALGEBRAS AND UNRANKED TREE LANGUAGES

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Abstract

In this paper we develop a variety theory for unranked tree languages and unranked algebras. In an unranked tree any symbol may label a node with any number of successors. Such trees appear in markup languages such as XML and as syntactic descriptions of natural languages. In the corresponding algebras each operation is defined for any number of arguments, but in the regular algebras used as tree recognizers the operations are finite-state computable. We develop the basic theory of regular algebras for a setting in which algebras over different operator alphabets are considered together. Using syntactic algebras of unranked tree languages we establish a bijection between varieties of unranked tree languages and varieties of regular algebras. As varieties of unranked tree languages are usually defined by means of congruences of term algebras, we introduce also varieties of congruences and a general device for defining such varieties. Finally, we show that the natural unranked counterparts of several varieties of ranked tree languages form varieties in our sense.

Keywords: unranked algebras, unranked trees, regular algebras, syntactic algebras, varieties of tree languages, varieties of regular algebras.

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REFERENCES

- [1] J. Almeida, *On pseudovarieties, varieties of languages, filters of congruences, pseudo-identities and related topics*, Algebra Universalis **27** (1990) 333–350. doi:10.1007/BF01190713
- [2] F. Baader and T. Nipkow, *Term Rewriting and All That* (Cambridge University Press, Cambridge, 1998). doi:10.1017/CBO9781139172752
- [3] C. Bergman, *Universal Algebra. Fundamentals and Selected Topics* (CRC Press, Taylor & Francis Group, Boca Raton, FL, 2012).
- [4] M. Bojańczyk, L. Segoufin and H. Straubing, *Piecewise testable tree languages*, Log. Methods in Comput. Sci. **8** (2012) 1–32. doi:10.2168/LMCS-8(3:26)2012
- [5] A. Brüggemann-Klein, M. Murata and D. Wood, *Regular tree and hedge languages over unranked alphabets*, Version 1, HKUST Theoretical Computer Science Center Research Report HKUST-TCS-2001-05 (Hong Kong, 2001).
- [6] S. Burris and H.P. Sankappanavar, *A Course in Universal Algebra* (Springer-Verlag, New York, 1981).
- [7] P.M. Cohn, *Universal Algebra*, 2. revised ed. (D. Reidel Publishing Company, Dordrecht, 1981).
- [8] H. Comon, M. Dauchet, R. Gilleron, C. Löding, F. Jacquemard, D. Lugiez, S. Tison and M. Tommasi, *Tree Automata Techniques and Applications*, available online: <http://www.grappa.univ-lille3.fr/tata>, 2007.
- [9] J. Cristau, C. Löding and W. Thomas, *Deterministic automata on unranked trees*, in: M. Liškiewicz, R. Reischuk (Eds.), *Fundamentals of Computation Theory, Proc. 15th Internat. Symp. FCT 2005, Lübeck 2005*, Lect. Notes Comput. Sci. 3623 (Springer, Berlin, 2005) pp. 68–79. doi:10.1007/11537311.7
- [10] K. Denecke and S.L. Wismath, *Universal Algebra and Applications in Theoretical Computer Science* (Chapman & Hall/CRC, Boca Raton, FL, 2002).
- [11] S. Eilenberg, *Automata, Languages, and Machines* (Academic Press, New York, Vol. A 1974, Vol. B 1976).
- [12] F. Gécseg and M. Steinby, *Tree Automata*, Akadémiai Kiadó, Budapest 1984. 2. ed. downloadable from arXiv.org as arXiv:1509.06233, September 2015.
- [13] F. Gécseg and M. Steinby, *Tree languages*, in: G. Rozenberg, A. Salomaa (Eds.), *Handbook of Formal Languages*, Vol. 3 (Springer, Berlin, 1997) pp. 1–69.
- [14] E. Jurvanen, A. Potthoff and W. Thomas, *Tree languages recognizable by regular frontier check*, in: G. Rozenberg, A. Salomaa (Eds.), *Developments in Language Theory*, World Scientific (Singapore, 1994) pp. 3–17.
- [15] J. Koppitz and K. Denecke, *M-Solid Varieties of Algebras* (Springer-Verlag, New York, 2006). doi:10.1007/0-387-30806-7
- [16] W. Martens and J. Niehren, *On the minimization of XML Schemas and tree automata for unranked trees*, J. Comput. Syst. Sci. **73** (2007) 550–583. doi:10.1016/j.jcss.2006.10.021

- [17] F. Neven, Automata, Logic, and XML, in: J. Bradfield (Ed.), Computer Science Logic, Proc. 16th Internat. Workshop, CSL 2002, Edinburgh, UK, 2002, Lect. Notes Comput. Sci. 2471 (Springer, Berlin, 2002) pp. 2–26.
- [18] C. Pair and A. Quere, *Définition et étude bilangages réguliers*, Inform. Control **13** (1968) 565–593.
doi:10.1016/S0019-9958(68)90999-6
- [19] V. Piirainen, Piecewise testable tree languages, TUCS Technical Report 634 (Turku Centre for Computer Science, Turku, 2004).
- [20] J.E. Pin, Varieties of Formal Languages (North Oxford Academic Publishers, London, 1986).
- [21] J. Sakarovitch, Elements of Automata Theory (Cambridge University Press, Cambridge, 2009).
- [22] T. Schwentick, *Automata for XML – A survey*, J. Comput. Syst. Sci. **73** (2007) 289–315.
doi:10.1016/j.jcss.2006.10.003
- [23] M. Steinby, *Syntactic algebras and varieties of recognizable sets*, in: M.C. Gaudel, J.P. Jouannaud (Eds.), Les Arbres en Algèbre et en Programmation, Proc. 4th CAAP (University of Lille, Lille, 1979) pp. 226–240.
- [24] M. Steinby, *A theory of tree language varieties*, in: M. Nivat, A. Podelski, (Eds.), Tree Automata and Languages (North-Holland, Amsterdam, 1992) pp. 57–81.
- [25] M. Steinby, *General varieties of tree languages*, Theor. Comput. Sci. **205** (1998) 1–43. doi:10.1016/S0304-3975(98)00010-3
- [26] M. Steinby, *Algebraic classifications of regular tree languages*, in: V.B. Kudryavtsev, I.G. Rosenberg (Eds.), Structural Theory of Automata, Semigroups, and Universal Algebra (Springer, Dordrecht, 2005) pp. 381–432.
- [27] M. Takahashi, *Generalizations of regular sets and their application to a study of context-free languages*, Inform. Control **27** (1975) 1–36.
doi:10.1016/S0019-9958(75)90058-3
- [28] J.W. Thatcher, *Characterizing derivation trees of context-free grammars through a generalization of finite automata theory*, J. Comput. Syst. Sci. **1** (1967) 317–322.
doi:10.1016/S0022-0000(67)80022-9
- [29] W. Thomas, *Logical aspects in the study of tree languages*, in: B. Courcelle (Ed.), 9th Colloquium on Trees in Algebra and Programming, Proc. Conf., Bordeaux, 1984 (Cambridge University Press, London, 1984) pp. 31–49.

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