

## ON THE GENUS OF THE CAYLEY GRAPH OF A COMMUTATIVE RING

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### Abstract

Let  $R$  be a commutative ring with non-zero identity and let  $Z(R)$  be the set of all zero-divisors. The Cayley graph  $\text{CAY}(R)$  of  $R$  is the simple undirected graph whose vertices are elements of  $R$  and two distinct vertices  $x$  and  $y$  are joined by an edge if and only if  $x - y \in Z(R)$ . In this paper, we determine all isomorphism classes of finite commutative rings with identity whose  $\text{CAY}(R)$  has genus one.

**Keywords:** Cayley graph, local ring, zero-divisor, planar graph, genus.

**2010 Mathematics Subject Classification:** 05C99, 05C15, 13A99.

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Received 22 April 2015

Revised 4 June 2016

