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QUASIORDER LATTICES ARE FIVE-GENERATED

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Abstract

A quasiorder (relation), also known as a preorder, is a reflexive and transitive relation. The quasiorders on a set A form a complete lattice with respect to set inclusion. Assume that A is a set such that there is no inaccessible cardinal less than or equal to $|A|$; note that in Kuratowski's model of ZFC, all sets A satisfy this assumption. Generalizing the 1996 result of Ivan Chajda and Gábor Czédli, also Tamás Dolgos' recent achievement, we prove that in this case the lattice of quasiorders on A is five-generated, as a complete lattice.

Keywords: quasiorder lattice, preorder lattice, accessible cardinal.

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