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APPLICATIONS OF SADDLE-POINT DETERMINANTS

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Abstract

For a given square matrix $\mathbf{A} \in M_n(\mathbb{R})$ and the vector $\mathbf{e} \in (\mathbb{R})^n$ of ones denote by $(\mathbf{A}, \mathbf{e})$ the matrix $\begin{bmatrix} \mathbf{A} & \mathbf{e} \\ \mathbf{e}^T & 0 \end{bmatrix}$.

This is often called the saddle point matrix and it plays a significant role in several branches of mathematics. Here we show some applications of it in: game theory and analysis. An application of specific saddle point matrices that are hollow, symmetric, and nonnegative is likewise shown in geometry as a generalization of Heron's formula to give the volume of a general simplex, as well as a conditions for its existence.

Keywords: bimatrix game, Mean Value Theorem, optimal mixed strategies, saddle point matrix, value of a game, volumes of simplices.

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