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A VARIATION OF ZERO-DIVISOR GRAPHS

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Abstract

In this paper, we define a new graph for a ring with unity by extending the definition of the usual *'zero-divisor graph'*. For a ring R with unity, $\Gamma_1(R)$ is defined to be the simple undirected graph having all non-zero elements of R as its vertices and two distinct vertices x, y are adjacent if and only if either $xy = 0$ or $yx = 0$ or $x + y$ is a unit. We consider the conditions of connectedness and show that for a finite commutative ring R with unity, $\Gamma_1(R)$ is connected if and only if R is not isomorphic to $\mathbb{Z}_3$ or $\mathbb{Z}_2^k$ (for any $k \in \mathbb{N} - \{1\}$). Then we characterize the rings R for which $\Gamma_1(R)$ realizes some well-known classes of graphs, viz., complete graphs, star graphs, paths (i.e., P_n), or cycles (i.e., C_n). We then look at different graph-theoretical properties of the graph $\Gamma_1(F)$, where F is a finite field. We also find all possible $\Gamma_1(R)$ graphs with at most 6 vertices.

Keywords: rings, zero-divisor graphs, finite fields.

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