

HOLOMORPH OF GENERALIZED BOL LOOPS II

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Abstract

The notion of the holomorph of a generalized Bol loop (GBL) is characterized afresh. The holomorph of a right inverse property loop (RIPL) is shown to be a GBL if and only if the loop is a GBL and some bijections of the loop are right (middle) regular. The holomorph of a RIPL is shown to be a GBL if and only if the loop is a GBL and some elements of the loop are right (middle) nuclear. Necessary and sufficient conditions for the holomorph of a RIPL to be a Bol loop are deduced. Some algebraic properties and commutative diagrams are established for a RIPL whose holomorph is a GBL.

Keywords: generalized Bol loop, holomorph of a loop.

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