

GRADED HILBERT-SYMBOL EQUIVALENCE OF NUMBER FIELDS

PRZEMYSŁAW KOPROWSKI

Faculty of Mathematics
University of Silesia
ul. Bankowa 14
40-007 Katowice, Poland

e-mail: pkoprowski@member.ams.org

Abstract

We present a new criterion for the existence of Hilbert-symbol equivalence of two number fields. In principle, we show that the system of local conditions for this equivalence may be expressed in terms of Clifford invariants in place of Hilbert-symbols, shifting the focus from Brauer groups to Brauer-Wall groups.

Keywords: Brauer group, Brauer-Wall group, Hilbert symbol equivalence, Witt equivalence, graded quaternion algebras.

2010 Mathematics Subject Classification: 11E81, 11E12.

REFERENCES

- [1] P.E. Conner, R. Perlis and K. Szymiczek, *Wild sets and 2-ranks of class groups*, *Acta Arith.* **79** (1) (1997) 83–91.
- [2] A. Czogała, *On reciprocity equivalence of quadratic number fields*, *Acta Arith.* **58** (1) (1991) 27–46.
- [3] A. Czogała, *Higher degree tame Hilbert-symbol equivalence of number fields*, *Abh. Math. Sem. Univ. Hamburg* **69** (1999) 175–185. doi:10.1007/BF02940871
- [4] A. Czogała, *Równoważność Hilberta ciał globalnych*, volume 1969 of *Prace Naukowe Uniwersytetu Śląskiego w Katowicach* [Scientific Publications of the University of Silesia], Wydawnictwo Uniwersytetu Śląskiego, Katowice, 2001.
- [5] A. Czogała and B. Rothkegel, *Wild primes of a self-equivalence of a number field*, *Acta Arith.* **166** (4) (2014) 335–348. doi:10.4064/aa166-4-2
- [6] A. Czogała and A. Śladek, *Higher degree Hilbert-symbol equivalence of number fields*, *Tatra Mt. Math. Publ.* **11** (1997) 77–88. Number theory (Liptovský Ján, 1995).

- [7] A. Czogała and A. Śladek, *Higher degree Hilbert symbol equivalence of algebraic number fields*, II, *J. Number Theory* **72** (2) (1998) 363–376. doi:10.1006/jnth.1998.2266
- [8] D.K. Harrison, *Witt Rings*, Lecture notes, Department of Mathematics, University of Kentucky (Lexington, Kentucky, 1970).
- [9] P. Koprowski, *Graded quaternion symbol equivalence of function fields*, *Czechoslovak Math. J.* **57** (132) (4) (2007), 1311–1319. doi:10.1007/s10587-007-0125-x
- [10] T.Y. Lam, *Introduction to Quadratic Forms Over Fields*, volume 67 of Graduate Studies in Mathematics, American Mathematical Society, Providence, RI, 2005.
- [11] T.C. Palfrey, *Density Theorems for Reciprocity Equivalences*, ProQuest LLC, Ann Arbor, MI, 1989. Thesis (Ph.D.)—Louisiana State University and Agricultural & Mechanical College.
- [12] R. Perlis, K. Szymiczek, P.E. Conner and R. Litherland, *Matching Witts with global fields*, in: *Recent advances in real algebraic geometry and quadratic forms* (Berkeley, CA, 1990/1991; San Francisco, CA, 1991), volume 155 of *Contemp. Math.*, pages 365–387. Amer. Math. Soc., Providence, RI, 1994.
- [13] A. Śladek, *Higher degree Harrison equivalence and Milnor K -functor*, in: *Proceedings of the 13th Czech and Slovak International Conference on Number Theory* (Ostravice, 1997), **6** (1998) 183–190.
- [14] M. Somodi, *On the size of the wild set*, *Canad. J. Math.* **57** (1) (2005) 180–203. doi:10.4153/CJM-2005-008-6
- [15] M. Somodi, *A characterization of the finite wild sets of rational self-equivalences*, *Acta Arith.* **121** (4) (2006) 327–334. doi:10.4064/aa121-4-3
- [16] K. Szymiczek, *Matching Witts locally and globally*, *Math. Slovaca* **41** (3) (1991) 315–330.
- [17] K. Szymiczek, *Witt equivalence of global fields*, *Comm. Algebra* **19** (4) (1991) 1125–1149.
- [18] K. Szymiczek, *Quadratic forms*, in: *Handbook of algebra*, Vol. 6, pages 35–80 (Elsevier/North-Holland, Amsterdam, 2009). doi:10.1016/S1570-7954(08)00202-7

Received 7 May 2015

Revised 8 June 2015