

JORDAN NUMBERS, STIRLING NUMBERS AND SUMS OF POWERS

ROMAN WITUŁA, KONRAD KACZMAREK, PIOTR LORENC,
EDYTA HETMANIOK AND MARIUSZ PLESZCZYŃSKI

Institute of Mathematics
Silesian University of Technology
Kaszubska 23, 44-100 Gliwice, Poland

e-mail: roman.witula,konrad.kaczmarek,edyta.hetmaniok@polsl.pl

Abstract

In the paper a new combinatorical interpretation of the Jordan numbers is presented. Binomial type formulae connecting both kinds of numbers mentioned in the title are given. The decomposition of the product of polynomial of variable n into the sums of k th powers of consecutive integers from 1 to n is also studied.

Keywords: Bernoulli numbers, binomial coefficients, Jordan numbers, Stirling numbers, Živković numbers.

2010 Mathematics Subject Classification: 11B68, 11B73, 11B83.

REFERENCES

- [1] Z.I. Borevich and I.R. Szafarevich, *Number Theory* (Nauka, Moscow, 1964, in Russian).
- [2] L. Carlitz, *Note on the numbers of Jordan and Ward*, *Duke Math. J.* **38** (1971) 783–790. doi:10.1215/S0012-7094-71-03894-4
- [3] L. Carlitz, *Some numbers related to the Stirling numbers of the first and second kind*, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz.* **544–576** (1976) 49–55.
- [4] L. Carlitz, *Some remarks on the Stirling numbers*, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz.* **678–715** (1980) 10–14.
- [5] K. Dilcher, *Bernoulli and Euler Polynomials*, 587–600 (in F.W.I. Olver, D.W. Lozier, R.F. Boisvert, C.W. Clark, *NIST Handbook of Mathematical Functions*, Cambridge Univ. Press, 2010).

- [6] R.L. Graham, D.E. Knuth and O. Patashnik, *Concrete Mathematics* (Addison-Wesley, Reading, 1994).
- [7] K. Ireland and M. Rosen, *A Classical Introduction to Modern Number Theory* (Springer, 1990). doi:10.1007/978-1-4757-1779-2
- [8] C. Jordan, *Calculus of Finite Differences* (Chelsea, New York, 1960). doi:10.2307/2333783
- [9] D.E. Knuth, *Johann Faulhaber and sums of powers*, *Math. Comp.* **203** (1993) 277–294. doi:10.2307/2152953
- [10] N. Nielsen, *Traité élémentaire des nombres de Bernoulli* (Gauthier – Villars, Paris, 1923).
- [11] S. Rabsztyn, D. Słota and R. Wituła, *Gamma and Beta Functions, Part I* (Silesian Technical University Press, Gliwice, 2011, in Polish).
- [12] J. Riordan, *An Introduction to Combinatorial Analysis* (John Wiley, 1958). doi:10.1063/1.3060724
- [13] J. Riordan, *Combinatorial Identities* (Wiley, New York, 1968).
- [14] N.J.A. Sloane, *The On-Line Encyclopedia of Integer Sequences* (<http://oeis.org/>).
- [15] M. Živković, *On a representation of Stirling's numbers of first kind*, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz.* **498–541** (1975) 217–221.

Received 6 December 2013

Revised 6 January 2015