

## SOME PROPERTIES OF THE ZERO DIVISOR GRAPH OF A COMMUTATIVE RING

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### Abstract

Let  $\Gamma(R)$  be the zero divisor graph for a commutative ring with identity. The  $k$ -domination number and the 2-packing number of  $\Gamma(R)$ , where  $R$  is an Artinian ring, are computed.  $k$ -dominating sets and 2-packing sets for the zero divisor graph of the ring of Gaussian integers modulo  $n$ ,  $\Gamma(\mathbb{Z}_n[i])$ , are constructed. The center, the median, the core, as well as the automorphism group of  $\Gamma(\mathbb{Z}_n[i])$  are determined. Perfect zero divisor graphs  $\Gamma(R)$  are investigated.

**Keywords:** automorphism group of a graph, center of a graph, core of a graph,  $k$ -domination number, Gaussian integers modulo  $n$ , median of a graph, 2-packing, perfect graph, and zero divisor graph.

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