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# SOME PROPERTIES OF THE ZERO DIVISOR GRAPH OF A COMMUTATIVE RING

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**Abstract**

Let $\Gamma(R)$ be the zero divisor graph for a commutative ring with identity. The $k$-domination number and the 2-packing number of $\Gamma(R)$, where $R$ is an Artinian ring, are computed. $k$-dominating sets and 2-packing sets for the zero divisor graph of the ring of Gaussian integers modulo $n$, $\Gamma(\mathbb{Z}_n[i])$, are constructed. The center, the median, the core, as well as the automorphism group of $\Gamma(\mathbb{Z}_n[i])$ are determined. Perfect zero divisor graphs $\Gamma(R)$ are investigated.

**Keywords:** automorphism group of a graph, center of a graph, core of a graph, $k$-domination number, Gaussian integers modulo $n$, median of a graph, 2-packing, perfect graph, and zero divisor graph.

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