

CHARACTERIZATIONS OF ORDERED Γ -ABEL-GRASSMANN'S GROUPOIDS

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Abstract

In this paper, we introduced the concept of ordered Γ -AG-groupoids, Γ -ideals and some classes in ordered Γ -AG-groupoids. We have shown that every Γ -ideal in an ordered Γ -AG^{**}-groupoid S is Γ -prime if and only if it is Γ -idempotent and the set of Γ -ideals of S is Γ -totally ordered under inclusion. We have proved that the set of Γ -ideals of S form a semilattice, also we have investigated some classes of ordered Γ -AG^{**}-groupoid and it has shown that weakly regular, intra-regular, right regular, left regular, left quasi regular, completely regular and (2, 2)-regular ordered Γ -AG^{**}-groupoids coincide. Further we have proved that every intra-regular ordered Γ -AG^{**}-groupoid is regular but the converse is not true in general. Furthermore we have shown that non-associative regular, weakly regular, intra-regular, right regular, left regular, left quasi regular, completely regular, (2, 2)-regular and strongly regular Γ -AG^{*}-groupoids do not exist.

Keywords: ordered Γ -AG-groupoids, Γ -ideals, regular Γ -AG^{**}-groupoids.

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