

## CHARACTERIZATIONS OF ORDERED $\Gamma$ -ABEL-GRASSMANN'S GROUPOIDS

MADAD KHAN, VENUS AMJID

*Department of Mathematics*  
*COMSATS Institute of Information Technology*  
*Abbottabad, Pakistan*

**e-mail:** madadmath@yahoo.com, venusmath@yahoo.com

GUL ZAMAN

*Department of Mathematics*  
*University of Malakand, Chakdara, Pakistan*

**e-mail:** gzaman@uom.edu.pk

AND

NAVEED YAQOOB

*Department of Mathematics*  
*Quaid-i-Azam University, Islamabad, Pakistan*

**e-mail:** nayaqoob@ymail.com

### Abstract

In this paper, we introduced the concept of ordered  $\Gamma$ -AG-groupoids,  $\Gamma$ -ideals and some classes in ordered  $\Gamma$ -AG-groupoids. We have shown that every  $\Gamma$ -ideal in an ordered  $\Gamma$ -AG<sup>\*\*</sup>-groupoid  $S$  is  $\Gamma$ -prime if and only if it is  $\Gamma$ -idempotent and the set of  $\Gamma$ -ideals of  $S$  is  $\Gamma$ -totally ordered under inclusion. We have proved that the set of  $\Gamma$ -ideals of  $S$  form a semilattice, also we have investigated some classes of ordered  $\Gamma$ -AG<sup>\*\*</sup>-groupoid and it has shown that weakly regular, intra-regular, right regular, left regular, left quasi regular, completely regular and (2, 2)-regular ordered  $\Gamma$ -AG<sup>\*\*</sup>-groupoids coincide. Further we have proved that every intra-regular ordered  $\Gamma$ -AG<sup>\*\*</sup>-groupoid is regular but the converse is not true in general. Furthermore we have shown that non-associative regular, weakly regular, intra-regular, right regular, left regular, left quasi regular, completely regular, (2, 2)-regular and strongly regular  $\Gamma$ -AG<sup>\*</sup>-groupoids do not exist.

**Keywords:** ordered  $\Gamma$ -AG-groupoids,  $\Gamma$ -ideals, regular  $\Gamma$ -AG<sup>\*\*</sup>-groupoids.

**2010 Mathematics Subject Classification:** 20M10, 20N99.

## REFERENCES

- [1] R. Chinram and K. Tinpun, *A note on minimal bi-ideals in ordered  $\Gamma$ -semigroups*, International Math. Forum **4** (1) (2009) 1–5.
- [2] K. Hila, *Filters in ordered  $\Gamma$ -semigroups*, Rocky Mountain J. Math. **41** (1) (2011) 189–203. doi:10.1216/RMJ-2011-41-1-189
- [3] K. Hila, *On quasi-prime, weakly quasi-prime left ideals in ordered  $\Gamma$ -semigroups*, Math. Slovaca **60** (2) (2010) 195–212. doi:10.2478/s12175-010-0006-x
- [4] K. Hila and E. Pisha, *On bi-ideals on ordered  $\Gamma$ -semigroups I*, Hacettepe J. Math. and Stat. **40** (6) (2011) 793–804.
- [5] K. Hila and E. Pisha, *On lattice-ordered rees matrix  $\Gamma$ -semigroups*, Annals of the Alexandru Ioan Cuza University - Mathematics **LIX** (1) (2013) 209–218. doi:10.2478/v10157-012-0033-8
- [6] K. Hila and E. Pisha, *Characterizations on ordered  $\Gamma$ -semigroups*, Inter. J. Pure and Appl. Math. **28** (3) (2006) 423–440.
- [7] P. Holgate, *Groupoids satisfying a simple invertive law*, The Math. Student **61** (1992) 101–106.
- [8] A. Iampan, *Characterizing ordered bi-Ideals in ordered  $\Gamma$ -semigroups*, Iranian J. Math. Sci. and Inf. **4** (1) (2009) 17–25.
- [9] A. Iampan, *Characterizing ordered quasi-ideals of ordered  $\Gamma$ -semigroups*, Kragujevac J. Math. **35** (1) (2011) 13–23.
- [10] M.A. Kazim and M. Naseeruddin, *On almost semigroups*, The Aligarh Bull. Math. **2** (1972) 1–7.
- [11] N. Kehayopulu and M. Tsingelis, *Regular ordered semigroups in terms of fuzzy subsets*, Inf. Sci. **176** (2006) 3675–3693. doi:10.1016/j.ins.2006.02.004
- [12] M. Khan, *Some studies in  $AG^*$ -groupoids*, Ph.D., thesis (Quaid-i-Azam University, Islamabad, Pakistan, 2008).
- [13] M. Khan and N. Ahmad, *Characterizations of left almost semigroups by their ideals*, J. Adv. Res. Pure Math. **2** (2010) 61–73. doi:10.5373/jarpm.357.020210
- [14] M. Khan, T. Asif and Faisal, *Intra-regular left almost semigroups characterized by their anti fuzzy ideals*, J. Math. Res. **4** (2010) 100–110.
- [15] Y.I. Kwon and S.K. Lee, *On weakly prime ideals of ordered  $\Gamma$ -semigroups*, Comm. Korean Math. Society **13** (2) (1998) 251–256.
- [16] Y.I. Kwon, *Characterizations of regular ordered  $\Gamma$ -semigroups II*, Far East J. Math. Sci. **11** (3) (2003) 281–287.
- [17] Y.I. Kwon and S.K. Lee, *Some special elements in ordered  $\Gamma$ -semigroups*, Kyungpook Math. J. **35** (1996) 679–685.
- [18] Y.I. Kwon and S.K. Lee, *The weakly semi-prime ideals of  $po$ - $\Gamma$ -semigroups*, Kangweon Kyungki Math. J. **5** (2) (1997) 135–139.

- [19] Q. Mushtaq and M. Khan, *Ideals in left almost semigroups*, Proceedings of 4th International Pure Mathematics Conference (2003) 65–77.
- [20] Q. Mushtaq and M.S. Kamran, *On LA-semigroups with weak associative law*, Scientific Khyber **1** (1989) 69–71.
- [21] Q. Mushtaq and S.M. Yousuf, *On LA-semigroups*, Aligarh Bull. Math. **8** (1978) 65–70.
- [22] Q. Mushtaq and S.M. Yousuf, *On LA-semigroup defined by a commutative inverse semigroup*, Math. Bech. **40** (1988) 59–62.
- [23] M. Naseeruddin, *Some studies in almost semigroups and flocks*, Ph.D., thesis (Aligarh Muslim University, Aligarh, India, 1970).
- [24] P.V. Protić and N. Stevanović, *AG-test and some general properties of Abel-Grassmann's groupoids*, Pure Math. and Appl. **4** (6) (1995) 371–383.
- [25] T. Shah and I. Rehman, *On M-systems in  $\Gamma$ -AG-groupoids*, Proceedings of the Pakistan Academy of Sciences **47** (1) (2010) 33–39.
- [26] N. Stevanović and P.V. Protić, *Composition of Abel-Grassmann's 3-bands*, Novi Sad J. Math. **2** (34) (2004) 175–182.

Received 9 August 2013  
Revised 24 January 2014