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CONGRUENCES AND BOOLEAN FILTERS OF QUASI-MODULAR p-ALGEBRAS

Abd El-Mohsen Badawy

Department of Mathematics
Faculty of Science
Tanta University, Tanta, Egypt

e-mail: abdelmohsen.badawy@yahoo.com

and

K.P. Shum

Institute of Mathematics
Yunnan University
Kunning, P.R. China

e-mail: kpshum@ynu.edu.cn

Abstract

The concept of Boolean filters in p-algebras is introduced. Some properties of Boolean filters are studied. It is proved that the class of all Boolean filters $BF(L)$ of a quasi-modular p-algebra L is a bounded distributive lattice. The Glivenko congruence Φ on a p-algebra L is defined by $(x, y) \in \Phi$ iff $x^{**} = y^{**}$. Boolean filters $[F_a), a \in B(L)$, generated by the Glivenko congruence classes F_a (where F_a is the congruence class $[a]\Phi$) are described in a quasi-modular p-algebra L. We observe that the set $F_B(L) = \{[F_a) : a \in B(L)\}$ is a Boolean algebra on its own. A one-one correspondence between the Boolean filters of a quasi-modular p-algebra L and the congruences in $[\Phi, \nabla]$ is established. Also some properties of congruences induced by the Boolean filters $[F_a), a \in B(L)$ are derived. Finally, we consider some properties of congruences with respect to the direct products of Boolean filters.

Keywords: p-algebras, quasi-modular p-algebras, Boolean filters, direct products, congruences.

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