

## A NOTE ON SEMIDIRECT SUM OF LIE ALGEBRAS

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### Abstract

In the paper there are investigated some properties of Lie algebras, the construction which has a wide range of applications like computer sciences (especially to computer visions), geometry or physics, for example. We concentrate on the semidirect sum of algebras and there are extended some theoretic designs as conditions to be a center, a homomorphism or a derivative. The Killing form of the semidirect sum where the second component is an ideal of the first one is considered as well.

**Keywords:** Lie algebra, subalgebra, ideal, center, semidirect sum, homomorphism, derivation, Killing form.

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