

## NIL-EXTENSIONS OF COMPLETELY SIMPLE SEMIRINGS

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### Abstract

A semiring  $S$  is said to be a quasi completely regular semiring if for any  $a \in S$  there exists a positive integer  $n$  such that  $na$  is completely regular. The present paper is devoted to the study of completely Archimedean semirings. We show that a semiring  $S$  is a completely Archimedean semiring if and only if it is a nil-extension of a completely simple semiring. This result extends the crucial structure theorem of completely Archimedean semigroup.

**Keywords:** ideal extension, nil-extension, bi-ideal, completely Archimedean semirings, completely simple semiring.

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