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ON RATIONAL RADII COIN REPRESENTATIONS OF THE WHEEL GRAPH

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Abstract

A *flower* is a coin graph representation of the wheel graph. A *petal* of a flower is an outer coin connected to the center coin. The results of this paper are twofold. First we derive a parametrization of all the rational (and hence integer) radii coins of the 3-petal flower, also known as Apollonian circles or Soddy circles. Secondly we consider a general n-petal flower and show there is a unique irreducible polynomial P_n in n variables over the rationals $\mathbb{Q}$, the affine variety of which contains the cosinus of the internal angles formed by the center coin and two consecutive petals of the flower. In that process we also derive a recursion that these irreducible polynomials satisfy.

Keywords: planar graph, coin graph, flower, polynomial ring, Galois theory.

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