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THE INERTIA OF UNICYCLIC GRAPHS AND BICYCLIC GRAPHS

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Abstract

Let G be a graph with n vertices and $\nu(G)$ be the matching number of G. The inertia of a graph G, $In(G) = (n_+, n_-, n_0)$ is an integer triple specifying the numbers of positive, negative and zero eigenvalues of the adjacency matrix $A(G)$, respectively. Let $\eta(G) = n_0$ denote the nullity of G (the multiplicity of the eigenvalue zero of G). It is well known that if G is a tree, then $\eta(G) = n - 2\nu(G)$. Guo *et al.* [Ji-Ming Guo, Weigen Yan and Yeong-Nan Yeh. On the nullity and the matching number of unicyclic graphs, Linear Algebra and its Applications, 431 (2009), 1293–1301.] proved if G is a unicyclic graph, then $\eta(G)$ equals $n - 2\nu(G) - 1, n - 2\nu(G)$ or $n - 2\nu(G) + 2$. Barrett *et al.* determined the inertia sets for trees and graphs with cut vertices. In this paper, we give the nullity of bicyclic graphs $\mathcal{B}_n^{++}$. Furthermore, we determine the inertia set in unicyclic graphs and $\mathcal{B}_n^{++}$, respectively.

Keywords: matching number, inertia, nullity, unicyclic graph, bicyclic graph.

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