

ON IDEALS OF A SKEW LATTICE

JOÃO PITA COSTA

University of Ljubljana

e-mail: joapitacosta@gmail.com

Abstract

Ideals are one of the main topics of interest when it comes to the study of the order structure of an algebra. Due to their nice properties, ideals have an important role both in lattice theory and semigroup theory. Two natural concepts of ideal can be derived, respectively, from the two concepts of order that arise in the context of skew lattices. The correspondence between the ideals of a skew lattice, derived from the preorder, and the ideals of its respective lattice image is clear. Though, skew ideals, derived from the partial order, seem to be closer to the specific nature of skew lattices. In this paper we review ideals in skew lattices and discuss the intersection of this with the study of the coset structure of a skew lattice.

Keywords: noncommutative lattice, skew lattice, band of semigroups, ideals, coset structure, Green's relations, skew Boolean algebras.

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