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LEAPING CONVERGENTS OF TASOEV CONTINUED FRACTIONS

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Abstract

Denote the n-th convergent of the continued fraction by $p_n/q_n = [a_0; a_1, \dots, a_n]$. We give some explicit forms of leaping convergents of Tasoev continued fractions. For instance, $[0; ua, ua^2, ua^3, \dots]$ is one of the typical types of Tasoev continued fractions. Leaping convergents are of the form p_{rn+i}/q_{rn+i} $(n = 0, 1, 2, \dots)$ for fixed integers $r \geq 2$ and $0 \leq i \leq r-1$.

Keywords: leaping convergents, Tasoev continued fractions.

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