

## ON FINITE FUNCTIONS WITH NON-TRIVIAL ARITY GAP

SLAVCHO SHTRAKOV

*Department of Computer Science*  
*South-West University, 2700 Blagoevgrad, Bulgaria*

**e-mail:** shtrakov@swu.bg

<http://home.swu.bg/shtrakov>

AND

JÖRG KOPPITZ\*

*Institute of Mathematics, University of Potsdam*  
*14415 Potsdam, Germany*

**e-mail:** koppitz@rz.uni-potsdam.de

### Abstract

Given an  $n$ -ary  $k$ -valued function  $f$ ,  $gap(f)$  denotes the minimal number of essential variables in  $f$  which become fictive when identifying any two distinct essential variables in  $f$ .

We particularly solve a problem concerning the explicit determination of  $n$ -ary  $k$ -valued functions  $f$  with  $2 \leq gap(f) \leq n \leq k$ . Our methods yield new combinatorial results about the number of such functions.

**Keywords:** essential variable, identification minor, essential arity gap.

**2000 Mathematics Subject Classification:** Primary: 03G25;  
Secondary: 05E05.

### REFERENCES

- [1] J. Berman and A. Kisielewicz, *On the number of operations in a clone*, Proc. Amer. Math Soc. **122** (1994), 359–369.  
doi:10.1090/S0002-9939-1994-1198450-9

---

\*The research was supported by the DFG project KO 1446/3-1.

- [2] Yu. Breitbart, *On the essential variables of functions in the algebra of logic*, Dokl. Acad. Sci. USSR, (in Russian) 172 vol. 1 (1967), 9–10 .
- [3] K. Chimev, *Separable sets of arguments of functions*, MTA SzTAKI Tanulmányok, 180 (1986), 173.
- [4] K. Chimev, *On some properties of functions*, Colloquia Mathematica Societatis Janos Bolyai, Szeged (1981), 97–110.
- [5] M. Couceiro and E. Lehtonen, *On the arity gap of finite functions: results and applications*, Int. Conf. on Relations, Orders and Graphs: Interaction with Computer Science, Nouha Editions, Sfax, (2008), pp. 65–72, (<http://www.math.tut.fi/algebra/papers/ROGICS08-CL.pdf>).
- [6] M. Couceiro and E. Lehtonen, *Generalizations of Swierczkowski’s lemma and the arity gap of finite functions*, Discrete Mathematics, (2009), doi:10.1016/j.disc.2009.04.009.
- [7] K. Denecke and J. Koppitz, *Essential variables in hypersubstitutions*, Algebra Universalis **46** (2001), 443–454. doi:10.1007/PL00000353
- [8] D. Kovachev, *On a class of discrete functions*, Acta Cybernetica, (Szeged) **17** (3) (2006), 513–519.
- [9] O. Lupanov, *On a class of schemes of functional elements*, Problemi Kybernetiki (in Russian) **9** (1963), 333–335.
- [10] A. Salomaa, *On essential variables of functions, especially in the algebra of logic*, Annales Academia Scientiarum Fennicae, Ser. A **333** (1963), 1–11.
- [11] Sl. Shtrakov and K. Denecke, *Essential variables and separable sets in universal algebra*, Taylor & Francis, Multiple-Valued Logic **8** (2) (2002), 165–182.
- [12] Sl. Shtrakov, *Essential variables and positions in terms*, Algebra Universalis **61** (3-4) (2009), 381–397. doi:10.1007/s00012-009-0023-1
- [13] Sl. Shtrakov, *Tree automata and essential input variables*, Contributions to General Algebra, Verlag Johannes Heyn, Klagenfurt **13** (2001), 309–320.
- [14] Sl. Shtrakov, *Essential arity gap of Boolean functions*, Serdica Journal of Computing **2** (3) (2008), 249–266.
- [15] R. Willard, *Essential arities of term operations in finite algebras*, Discrete Mathematics **149** (1996), 239–259. doi:10.1016/0012-365X(94)00323-B

Received 15 December 2009

Revised 14 January 2010