

THE MONOID OF GENERALIZED
HYPERSUBSTITUTIONS OF TYPE $\tau = (n)$

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Abstract

A (usual) hypersubstitution of type τ is a function which takes each operation symbol of the type to a term of the type, of the same arity. The set of all hypersubstitutions of a fixed type τ forms a monoid under composition, and semigroup properties of this monoid have been studied by a number of authors. In particular, idempotent and regular elements, and the Green's relations, have been studied for type (n) by S.L. Wismath.

A generalized hypersubstitution of type $\tau = (n)$ is a mapping σ which takes the n -ary operation symbol f to a term $\sigma(f)$ which does not necessarily preserve the arity. Any such σ can be inductively extended to a map $\hat{\sigma}$ on the set of all terms of type $\tau = (n)$, and any two such extensions can be composed in a natural way. Thus, the set $Hyp_G(n)$ of all generalized hypersubstitutions of type $\tau = (n)$ forms a monoid. In this paper we study the semigroup properties of $Hyp_G(n)$.

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In particular, we characterize the idempotent and regular generalized hypersubstitutions, and describe some classes under Green's relations of this monoid.

Keywords: monoid, regular elements, idempotent elements, Green's relations, generalized hypersubstitution.

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