

## A REDUCTION THEOREM FOR RING VARIETIES WHOSE SUBVARIETY LATTICE IS DISTRIBUTIVE

MIKHAIL V. VOLKOV\*

*Faculty of Mathematics and Mechanics*  
*Ural State University*  
*Lenina 51, 620083 Ekaterinburg, Russia*

**e-mail:** Mikhail.Volkov@usu.ru

### Abstract

We prove a theorem (for arbitrary ring varieties and, in a stronger form, for varieties of associative rings) which basically reduces the problem of a description of varieties with distributive subvariety lattice to the case of algebras over a finite prime field.

**Keywords:** variety of rings, subvariety lattice, distributive lattice, torsion-bounded variety, Mal'tsev product.

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